Proof of Theorem 154

The theorem to be proved is

$x \leq y \quad \rightarrow \quad x \leq \mathrm{S}y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \leq (y)] \quad \& \quad [\neg\, (x) \leq (\mathrm{S}y)]]$

Special cases of the hypothesis and previous results:

0: $x \leq y$ from H:x:y

1: $\neg\, x \leq \mathrm{S}y$ from H:x:y

2: $y \leq \mathrm{S}y$ from 63;y

3: $\neg\, x \leq y \quad \vee \quad \neg\, y \leq \mathrm{S}y \quad \vee \quad x \leq \mathrm{S}y$ from 73;x;y;Sy

Inferences:

4: $\neg\, y \leq \mathrm{S}y \quad \vee \quad x \leq \mathrm{S}y$ by

0: $x \leq y$

3: $\neg\, x \leq y \quad \vee \quad \neg\, y \leq \mathrm{S}y \quad \vee \quad x \leq \mathrm{S}y$

5: $\neg\, y \leq \mathrm{S}y$ by

1: $\neg\, x \leq \mathrm{S}y$

4: $\neg\, y \leq \mathrm{S}y \quad \vee \quad x \leq \mathrm{S}y$

6: QEA by

2: $y \leq \mathrm{S}y$

5: $\neg\, y \leq \mathrm{S}y$