Proof of Theorem 152

The theorem to be proved is

$\mathrm{p}_{150}(q, x) \quad \& \quad \mathrm{SS}x < 2 \cdot q \quad \rightarrow \quad \mathrm{p}_{150}(q, \mathrm{S}x)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\mathrm{p}_{150}((qx))] \quad \& \quad [(\mathrm{S}(\mathrm{S}x)) < (2 \cdot q)] \quad \& \quad [\neg\, \mathrm{p}_{150}((q\mathrm{S}x))]]$

Special cases of the hypothesis and previous results:

0: $\mathrm{p}_{150}((qx))$ from H:q:x

1: $\mathrm{S}(\mathrm{S}x) < 2 \cdot q$ from H:q:x

2: $\neg\, \mathrm{p}_{150}((q\mathrm{S}x))$ from H:q:x

3: $\neg\, \mathrm{p}_{150}((qx))$ $\vee$ q is a power of two from 150$^{\rightarrow}$;q;x

4: $\neg\, \mathrm{p}_{150}((qx)) \vee q \leq \mathrm{S}x$ from 150$^{\rightarrow}$;q;x

5: $\mathrm{p}_{150}((q\mathrm{S}x))$ $\vee$ $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x) \vee \neg\, \mathrm{S}(\mathrm{S}x) < 2 \cdot q$ from 150$^{\leftarrow}$;q;$\mathrm{S}x$

6: $\mathrm{S}x \leq \mathrm{S}(\mathrm{S}x)$ from 63;$\mathrm{S}x$

7: $\neg\, q \leq \mathrm{S}x \vee \neg\, \mathrm{S}x \leq \mathrm{S}(\mathrm{S}x) \vee q \leq \mathrm{S}(\mathrm{S}x)$ from 73;q;$\mathrm{S}x$;$\mathrm{S}(\mathrm{S}x)$

Inferences:

8: q is a power of two by

0: $\mathrm{p}_{150}((qx))$

3: $\neg\, \mathrm{p}_{150}((qx))$ $\vee$ q is a power of two

9: $q \leq \mathrm{S}x$ by

0: $\mathrm{p}_{150}((qx))$

4: $\neg\, \mathrm{p}_{150}((qx)) \vee q \leq \mathrm{S}x$

10: $\mathrm{p}_{150}((q\mathrm{S}x))$ $\vee$ $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$ by

1: $\mathrm{S}(\mathrm{S}x) < 2 \cdot q$

5: $\mathrm{p}_{150}((q\mathrm{S}x))$ $\vee$ $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x) \vee \neg\, \mathrm{S}(\mathrm{S}x) < 2 \cdot q$

11: $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$ by

2: $\neg\, \mathrm{p}_{150}((q\mathrm{S}x))$

10: $\mathrm{p}_{150}((q\mathrm{S}x))$ $\vee$ $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$

12: $\neg\, q \leq \mathrm{S}x \quad \vee \quad q \leq \mathrm{S}(\mathrm{S}x)$ by

6: $\mathrm{S}x \leq \mathrm{S}(\mathrm{S}x)$

7: $\neg\, q \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x \leq \mathrm{S}(\mathrm{S}x) \quad \vee \quad q \leq \mathrm{S}(\mathrm{S}x)$

13: $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$ by

8: q is a power of two

11: $\neg\, q$ is a power of two $\vee$ $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$

14: $q \leq \mathrm{S}(\mathrm{S}x)$ by

9: $q \leq \mathrm{S}x$

12: $\neg\, q \leq \mathrm{S}x \quad \vee \quad q \leq \mathrm{S}(\mathrm{S}x)$

15: QEA by

13: $\neg\, q \leq \mathrm{S}(\mathrm{S}x)$

14: $q \leq \mathrm{S}(\mathrm{S}x)$