Proof of Theorem 151a

The theorem to be proved is

$\mathrm{p}_{150}(q, x) \quad \& \quad \mathrm{p}_{150}(q', x) \quad \rightarrow \quad \neg\, q < q'$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $'[[\mathrm{p}_{150}((qx))] \quad \& \quad [\mathrm{p}_{150}((q'x))] \quad \& \quad [(q) < (q')]]$

Special cases of the hypothesis and previous results:

0: $\mathrm{p}_{150}((qx))$ from H:q:x:q'

1: $\mathrm{p}_{150}((q'x))$ from H:q:x:q'

2: $q < q'$ from H:q:x:q'

3: $\neg\, \mathrm{p}_{150}((qx)) \quad \vee \quad q$ is a power of two from 150$^{\rightarrow}$;q;x

4: $\neg\, \mathrm{p}_{150}((qx)) \quad \vee \quad \mathrm{S}x < 2 \cdot q$ from 150$^{\rightarrow}$;q;x

5: $\neg\, \mathrm{p}_{150}((q'x)) \quad \vee \quad q'$ is a power of two from 150$^{\rightarrow}$;q';x

6: $\neg\, \mathrm{p}_{150}((q'x)) \quad \vee \quad q' \leq \mathrm{S}x$ from 150$^{\rightarrow}$;q';x

7: $\neg\, q$ is a power of two $\vee$ $\neg\, q'$ is a power of two $\vee$ $\neg\, q < q'$ $\vee$ $\neg\, q' < 2 \cdot q$ from 149;q;q'

8: $\neg\, q' \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q \quad \vee \quad q' < 2 \cdot q$ from 122;q';Sx;$2 \cdot q$

Inferences:

9: q is a power of two by

0: $\mathrm{p}_{150}((qx))$

3: $\neg\, \mathrm{p}_{150}((qx)) \quad \vee \quad q$ is a power of two

10: $\mathrm{S}x < 2 \cdot q$ by

0: $\mathrm{p}_{150}((qx))$

4: $\neg\, \mathrm{p}_{150}((qx)) \quad \vee \quad \mathrm{S}x < 2 \cdot q$

11: q' is a power of two by

1: $\mathrm{p}_{150}((q'x))$

5: $\neg\, \mathrm{p}_{150}((q'x)) \quad \vee \quad q'$ is a power of two

12: $q' \leq \mathrm{S}x$ by

1: $\mathrm{p}_{150}((q'x))$

6: $\neg\, \mathrm{p}_{150}((q'x)) \quad \vee \quad q' \leq \mathrm{S}x$

13: $\neg\, q$ is a power of two $\vee$ $\neg\, q'$ is a power of two $\vee$ $\neg\, q' < 2 \cdot q$ by

2: $q < q'$

7: $\neg\, q$ is a power of two $\vee$ $\neg\, q'$ is a power of two $\vee$ $\neg\, q < q'$ $\vee$ $\neg\, q' < 2 \cdot q$

14: $\neg\, q'$ is a power of two $\vee$ $\neg\, q' < 2 \cdot q$ by

9: q is a power of two

13: $\neg\, q$ is a power of two $\vee$ $\neg\, q'$ is a power of two $\vee$ $\neg\, q' < 2 \cdot q$

15: $\neg\, q' \leq \mathrm{S}x$ $\vee$ $q' < 2 \cdot q$ by

10: $\mathrm{S}x < 2 \cdot q$

8: $\neg\, q' \leq \mathrm{S}x$ $\vee$ $\neg\, \mathrm{S}x < 2 \cdot q$ $\vee$ $q' < 2 \cdot q$

16: $\neg\, q' < 2 \cdot q$ by

11: q' is a power of two

14: $\neg\, q'$ is a power of two $\vee$ $\neg\, q' < 2 \cdot q$

17: $q' < 2 \cdot q$ by

12: $q' \leq \mathrm{S}x$

15: $\neg\, q' \leq \mathrm{S}x$ $\vee$ $q' < 2 \cdot q$

18: QEA by

16: $\neg\, q' < 2 \cdot q$

17: $q' < 2 \cdot q$