Proof of Theorem 151

The theorem to be proved is

$p_{150}(q,x) \quad \& \quad p_{150}(q',x) \quad \rightarrow \quad q = q'$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\;'[[p_{150}((qx))] \quad \& \quad [p_{150}((q'x))] \quad \& \quad [\neg\,(q) = (q')]]$

Special cases of the hypothesis and previous results:

0: $p_{150}((qx))$ from H:q:x:q'

1: $p_{150}((q'x))$ from H:q:x:q'

2: $\neg\, q' = q$ from H:q:x:q'

3: $\neg\, p_{150}((qx)) \quad \vee \quad \neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q < q'$ from 151a;q;x;q'

4: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, p_{150}((qx)) \quad \vee \quad \neg\, q' < q$ from 151a;q';x;q

5: $q < q' \quad \vee \quad q' = q \quad \vee \quad q' < q$ from 77;q;q'

Inferences:

6: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q < q'$ by
0: $p_{150}((qx))$
3: $\neg\, p_{150}((qx)) \quad \vee \quad \neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q < q'$

7: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q' < q$ by
0: $p_{150}((qx))$
4: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, p_{150}((qx)) \quad \vee \quad \neg\, q' < q$

8: $\neg\, q < q'$ by
1: $p_{150}((q'x))$
6: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q < q'$

9: $\neg\, q' < q$ by
1: $p_{150}((q'x))$
7: $\neg\, p_{150}((q'x)) \quad \vee \quad \neg\, q' < q$

10: $q < q' \quad \vee \quad q' < q$ by
2: $\neg\, q' = q$
5: $q < q' \quad \vee \quad q' = q \quad \vee \quad q' < q$

11: $q' < q$ by

8: $\neg\ q < q'$

10: $q < q' \quad \vee \quad q' < q$

12: QEA by

9: $\neg\ q' < q$

11: $q' < q$