Proof of Theorem 14b

The theorem to be proved is

$x + \mathrm{S}0 = \mathrm{S}x + 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x + (\mathrm{S}0)) = ((\mathrm{S}x) + 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (\mathrm{S}x) + 0 = x + (\mathrm{S}0)$ from H:x

1: $x + 0 = x$ from 12;x;0

2: $\mathrm{S}(x + 0) = x + (\mathrm{S}0)$ from 12;x;0

3: $(\mathrm{S}x) + 0 = \mathrm{S}x$ from 12;Sx

Equality substitutions:

4: $\neg\, x + 0 = x \quad \lor \quad \neg\, \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \lor \quad \mathrm{S}(x) = x + (\mathrm{S}0)$

5: $\neg\, (\mathrm{S}x) + 0 = \mathrm{S}x \quad \lor \quad (\mathrm{S}x) + 0 = x + (\mathrm{S}0) \quad \lor \quad \neg\, \mathrm{S}x = x + (\mathrm{S}0)$

Inferences:

6: $\neg\, (\mathrm{S}x) + 0 = \mathrm{S}x \quad \lor \quad \neg\, x + (\mathrm{S}0) = \mathrm{S}x$ by
0: $\neg\, (\mathrm{S}x) + 0 = x + (\mathrm{S}0)$
5: $\neg\, (\mathrm{S}x) + 0 = \mathrm{S}x \quad \lor \quad (\mathrm{S}x) + 0 = x + (\mathrm{S}0) \quad \lor \quad \neg\, x + (\mathrm{S}0) = \mathrm{S}x$

7: $\neg\, \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \lor \quad x + (\mathrm{S}0) = \mathrm{S}x$ by
1: $x + 0 = x$
4: $\neg\, x + 0 = x \quad \lor \quad \neg\, \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \lor \quad x + (\mathrm{S}0) = \mathrm{S}x$

8: $x + (\mathrm{S}0) = \mathrm{S}x$ by
2: $\mathrm{S}(x + 0) = x + (\mathrm{S}0)$
7: $\neg\, \mathrm{S}(x + 0) = x + (\mathrm{S}0) \quad \lor \quad x + (\mathrm{S}0) = \mathrm{S}x$

9: $\neg\, x + (\mathrm{S}0) = \mathrm{S}x$ by
3: $(\mathrm{S}x) + 0 = \mathrm{S}x$
6: $\neg\, (\mathrm{S}x) + 0 = \mathrm{S}x \quad \lor \quad \neg\, x + (\mathrm{S}0) = \mathrm{S}x$

10: QEA by
8: $x + (\mathrm{S}0) = \mathrm{S}x$
9: $\neg\, x + (\mathrm{S}0) = \mathrm{S}x$