

Proof of Theorem 148a

The theorem to be proved is

$$2 \uparrow x = 2 \uparrow y \rightarrow \neg x < y$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[(2 \uparrow x) = (2 \uparrow y)] \ \& \ [(x) < (y)]]$$

Special cases of the hypothesis and previous results:

- 0: $2 \uparrow y = 2 \uparrow x$ from $H:x:y$
- 1: $x < y$ from $H:x:y$
- 2: $\neg x < y \vee 2 \uparrow x < 2 \uparrow y$ from [145](#); $x;y$
- 3: $\neg 2 \uparrow x < 2 \uparrow y \vee \neg 2 \uparrow y = 2 \uparrow x$ from [56](#)^{->}; $2 \uparrow x; 2 \uparrow y$

Inferences:

- 4: $\neg 2 \uparrow x < 2 \uparrow y$ by
 - 0: $2 \uparrow y = 2 \uparrow x$
 - 3: $\neg 2 \uparrow x < 2 \uparrow y \vee \neg 2 \uparrow y = 2 \uparrow x$
- 5: $2 \uparrow x < 2 \uparrow y$ by
 - 1: $x < y$
 - 2: $\neg x < y \vee 2 \uparrow x < 2 \uparrow y$
- 6: *QEA* by
 - 4: $\neg 2 \uparrow x < 2 \uparrow y$
 - 5: $2 \uparrow x < 2 \uparrow y$