Proof of Theorem 148

The theorem to be proved is

$2 \uparrow x = 2 \uparrow y \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(2 \uparrow x) = (2 \uparrow y)] \quad \& \quad [\neg\, (x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $2 \uparrow y = 2 \uparrow x$ from H:x:y

1: $\neg\, y = x$ from H:x:y

2: $\neg\, 2 \uparrow y = 2 \uparrow x \quad \vee \quad \neg\, x < y$ from 148a;x;y

3: $\neg\, 2 \uparrow y = 2 \uparrow x \quad \vee \quad \neg\, y < x$ from 148a;y;x

4: $x < y \quad \vee \quad y = x \quad \vee \quad y < x$ from 77;x;y

Inferences:

5: $\neg\, x < y$ by
0: $2 \uparrow y = 2 \uparrow x$
2: $\neg\, 2 \uparrow y = 2 \uparrow x \quad \vee \quad \neg\, x < y$

6: $\neg\, y < x$ by
0: $2 \uparrow y = 2 \uparrow x$
3: $\neg\, 2 \uparrow y = 2 \uparrow x \quad \vee \quad \neg\, y < x$

7: $x < y \quad \vee \quad y < x$ by
1: $\neg\, y = x$
4: $x < y \quad \vee \quad y = x \quad \vee \quad y < x$

8: $y < x$ by
5: $\neg\, x < y$
7: $x < y \quad \vee \quad y < x$

9: QEA by
6: $\neg\, y < x$
8: $y < x$