Proof of Theorem 143

The theorem to be proved is

$x \neq 0 \quad \rightarrow \quad x < x \cdot 2$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) = (0)] \quad \& \quad [\neg\,(x) < (x \cdot 2)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 = x$ from H:x

1: $\neg\, x < x \cdot 2$ from H:x

2: $0 = x \quad \vee \quad x < 2 \cdot x$ from 137;x

3: $2 \cdot x = x \cdot 2$ from 105;x;2

Equality substitutions:

4: $\neg\, 2 \cdot x = x \cdot 2 \quad \vee \quad \neg\, x < 2 \cdot x \quad \vee \quad x < x \cdot 2$

Inferences:

5: $x < 2 \cdot x$ by
0: $\neg\, 0 = x$
2: $0 = x \quad \vee \quad x < 2 \cdot x$

6: $\neg\, 2 \cdot x = x \cdot 2 \quad \vee \quad \neg\, x < 2 \cdot x$ by
1: $\neg\, x < x \cdot 2$
4: $\neg\, 2 \cdot x = x \cdot 2 \quad \vee \quad \neg\, x < 2 \cdot x \quad \vee \quad x < x \cdot 2$

7: $\neg\, x < 2 \cdot x$ by
3: $2 \cdot x = x \cdot 2$
6: $\neg\, 2 \cdot x = x \cdot 2 \quad \vee \quad \neg\, x < 2 \cdot x$

8: QEA by
5: $x < 2 \cdot x$
7: $\neg\, x < 2 \cdot x$