Proof of Theorem 141a

The theorem to be proved is

$x \cdot z = y \cdot z \quad \& \quad x < y \quad \& \quad z \neq 0 \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \cdot z) = (y \cdot z)] \quad \& \quad [(x) < (y)] \quad \& \quad [\neg (z) = (0)] \quad \& \quad [\neg (x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $y \cdot z = x \cdot z$ from H:x:z:y

1: $x < y$ from H:x:z:y

2: $\neg\, 0 = z$ from H:x:z:y

3: $\neg\, x < y \quad \vee \quad x + (y - x) = y$ from 108;x;y

4: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$ from 108;x;y

5: $(x \cdot z) + ((y - x) \cdot z) = (x + (y - x)) \cdot z$ from 106;x;$y - x$;z

6: $\neg\, (y - x) \cdot z = 0 \quad \vee \quad y - x = 0 \quad \vee \quad 0 = z$ from 132;$y - x$;z

7: $(x \cdot z) + 0 = x \cdot z$ from 12;$x \cdot z$

8: $\neg\, (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0 \quad \vee \quad (y - x) \cdot z = 0$ from 120;$x \cdot z$;0;$(y - x) \cdot z$

Equality substitutions:

9: $\neg\, y \cdot z = x \cdot z \quad \vee \quad (x \cdot z) + 0 = y \cdot z \quad \vee \quad \neg\, (x \cdot z) + 0 = x \cdot z$

10: $\neg\, x + (y - x) = y \quad \vee \quad \neg\, (x \cdot z) + ((y - x) \cdot z) = (x + (y - x)) \cdot z \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = (y) \cdot z$

11: $\neg\, (x \cdot z) + ((y - x) \cdot z) = y \cdot z \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0 \quad \vee \quad \neg\, y \cdot z = (x \cdot z) + 0$

Inferences:

12: $(x \cdot z) + 0 = y \cdot z \quad \vee \quad \neg\, (x \cdot z) + 0 = x \cdot z$ by

0: $y \cdot z = x \cdot z$

9: $\neg\, y \cdot z = x \cdot z \quad \vee \quad (x \cdot z) + 0 = y \cdot z \quad \vee \quad \neg\, (x \cdot z) + 0 = x \cdot z$

13: $x + (y - x) = y$ by

1: $x < y$

3: $\neg\, x < y \quad \vee \quad x + (y - x) = y$

14: $\neg\, y - x = 0$ by

1: $x < y$

4: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$

15: $\neg\, (y - x) \cdot z = 0 \quad \vee \quad y - x = 0$ by

2: $\neg\, 0 = z$

6: $\neg\, (y - x) \cdot z = 0 \quad \vee \quad y - x = 0 \quad \vee \quad 0 = z$

16: $\neg\, x + (y - x) = y \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = y \cdot z$ by

5: $(x \cdot z) + ((y - x) \cdot z) = (x + (y - x)) \cdot z$

10: $\neg\, x + (y - x) = y \quad \vee \quad \neg\, (x \cdot z) + ((y - x) \cdot z) = (x + (y - x)) \cdot z \quad \vee$ $(x \cdot z) + ((y - x) \cdot z) = y \cdot z$

17: $(x \cdot z) + 0 = y \cdot z$ by

7: $(x \cdot z) + 0 = x \cdot z$

12: $(x \cdot z) + 0 = y \cdot z \quad \vee \quad \neg\, (x \cdot z) + 0 = x \cdot z$

18: $(x \cdot z) + ((y - x) \cdot z) = y \cdot z$ by

13: $x + (y - x) = y$

16: $\neg\, x + (y - x) = y \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = y \cdot z$

19: $\neg\, (y - x) \cdot z = 0$ by

14: $\neg\, y - x = 0$

15: $\neg\, (y - x) \cdot z = 0 \quad \vee \quad y - x = 0$

20: $\neg\, (x \cdot z) + ((y - x) \cdot z) = y \cdot z \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$ by

17: $(x \cdot z) + 0 = y \cdot z$

11: $\neg\, (x \cdot z) + ((y - x) \cdot z) = y \cdot z \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$ $\vee \quad \neg\, (x \cdot z) + 0 = y \cdot z$

21: $(x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$ by

18: $(x \cdot z) + ((y - x) \cdot z) = y \cdot z$

20: $\neg\, (x \cdot z) + ((y - x) \cdot z) = y \cdot z \quad \vee \quad (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$

22: $\neg\, (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$ by

19: $\neg\, (y - x) \cdot z = 0$

8: $\neg\, (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0 \quad \vee \quad (y - x) \cdot z = 0$

23: QEA by

21: $(x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$

22: $\neg\, (x \cdot z) + ((y - x) \cdot z) = (x \cdot z) + 0$