Proof of Theorem 141

The theorem to be proved is

$x \cdot z = y \cdot z \quad \& \quad z \neq 0 \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \cdot z) = (y \cdot z)] \quad \& \quad [\neg\, (z) = (0)] \quad \& \quad [\neg\, (x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $y \cdot z = x \cdot z$ from H;x;z;y

1: $\neg\, 0 = z$ from H;x;z;y

2: $\neg\, y = x$ from H;x;z;y

3: $\neg\, y \cdot z = x \cdot z \quad \vee \quad \neg\, x < y \quad \vee \quad 0 = z \quad \vee \quad y = x$ from 141a;x;z;y

4: $\neg\, y \cdot z = x \cdot z \quad \vee \quad \neg\, y < x \quad \vee \quad 0 = z \quad \vee \quad y = x$ from 141a;y;z;x

5: $x < y \quad \vee \quad y = x \quad \vee \quad y < x$ from 77;x;y

Inferences:

6: $\neg\, x < y \quad \vee \quad 0 = z \quad \vee \quad y = x$ by

0: $y \cdot z = x \cdot z$

3: $\neg\, y \cdot z = x \cdot z \quad \vee \quad \neg\, x < y \quad \vee \quad 0 = z \quad \vee \quad y = x$

7: $\neg\, y < x \quad \vee \quad 0 = z \quad \vee \quad y = x$ by

0: $y \cdot z = x \cdot z$

4: $\neg\, y \cdot z = x \cdot z \quad \vee \quad \neg\, y < x \quad \vee \quad 0 = z \quad \vee \quad y = x$

8: $\neg\, x < y \quad \vee \quad y = x$ by

1: $\neg\, 0 = z$

6: $\neg\, x < y \quad \vee \quad 0 = z \quad \vee \quad y = x$

9: $\neg\, y < x \quad \vee \quad y = x$ by

1: $\neg\, 0 = z$

7: $\neg\, y < x \quad \vee \quad 0 = z \quad \vee \quad y = x$

10: $x < y \quad \vee \quad y < x$ by

2: $\neg\, y = x$

5: $x < y \quad \vee \quad y = x \quad \vee \quad y < x$

11: $\neg\, x < y$ by

2: $\neg\, y = x$

8: $\neg\, x < y \quad \vee \quad y = x$

12: $\neg\, y < x$ by

2: $\neg\, y = x$

9: $\neg\, y < x \quad \vee \quad y = x$

13: $y < x$ by

11: $\neg\, x < y$

10: $x < y \quad \vee \quad y < x$

14: QEA by

12: $\neg\, y < x$

13: $y < x$