Proof of Theorem 138

The theorem to be proved is

$0 \leq x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (0) \leq (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 \leq x$ from H:x

1: $0 + x = x$ from 97;x

2: $0 \leq 0 + x$ from 71;0;x

Equality substitutions:

3: $\neg\, 0 + x = x \quad \vee \quad \neg\, 0 \leq 0 + x \quad \vee \quad 0 \leq x$

Inferences:

4: $\neg\, 0 + x = x \quad \vee \quad \neg\, 0 \leq 0 + x$ by

0: $\neg\, 0 \leq x$

3: $\neg\, 0 + x = x \quad \vee \quad \neg\, 0 \leq 0 + x \quad \vee \quad 0 \leq x$

5: $\neg\, 0 \leq 0 + x$ by

1: $0 + x = x$

4: $\neg\, 0 + x = x \quad \vee \quad \neg\, 0 \leq 0 + x$

6: QEA by

2: $0 \leq 0 + x$

5: $\neg\, 0 \leq 0 + x$