Proof of Theorem 137

The theorem to be proved is

$x \neq 0 \quad \rightarrow \quad x < 2 \cdot x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x) = (0)] \quad \& \quad [\neg\ (x) < (2 \cdot x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ 0 = x$ from H:x

1: $\neg\ x < 2 \cdot x$ from H:x

2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$ from 22;x

3: $\mathrm{P}x < \mathrm{S}(\mathrm{P}x)$ from 125;$\mathrm{P}x$

4: $\neg\ \mathrm{P}x < x \quad \vee \quad \mathrm{S}(\mathrm{P}x) < 2 \cdot x$ from 124;$\mathrm{P}x$;x

Equality substitutions:

5: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{P}x < \mathrm{S}(\mathrm{P}x) \quad \vee \quad \mathrm{P}x < x$

6: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x \quad \vee \quad x < 2 \cdot x$

Inferences:

7: $\mathrm{S}(\mathrm{P}x) = x$ by
0: $\neg\ 0 = x$
2: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

8: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x$ by
1: $\neg\ x < 2 \cdot x$
6: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x \quad \vee \quad x < 2 \cdot x$

9: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{P}x < x$ by
3: $\mathrm{P}x < \mathrm{S}(\mathrm{P}x)$
5: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{P}x < \mathrm{S}(\mathrm{P}x) \quad \vee \quad \mathrm{P}x < x$

10: $\neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x$ by
7: $\mathrm{S}(\mathrm{P}x) = x$
8: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x$

11: $\mathrm{P}x < x$ by

7: $\mathrm{S}(\mathrm{P}x) = x$

9: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{P}x < x$

12: $\neg\ \mathrm{P}x < x$ by

10: $\neg\ \mathrm{S}(\mathrm{P}x) < 2 \cdot x$

4: $\neg\ \mathrm{P}x < x \quad \vee \quad \mathrm{S}(\mathrm{P}x) < 2 \cdot x$

13: QEA by

11: $\mathrm{P}x < x$

12: $\neg\ \mathrm{P}x < x$