Proof of Theorem 136b

The theorem to be proved is

$x \uparrow (y + 0) = (x \uparrow y) \cdot (x \uparrow 0)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x \uparrow (y + 0)) = ((x \uparrow y) \cdot (x \uparrow 0))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (x \uparrow y) \cdot (x \uparrow 0) = x \uparrow (y + 0)$ from H:x:y

1: $y + 0 = y$ from 12;y

2: $x \uparrow 0 = 1$ from 126;x

3: $1 \cdot (x \uparrow y) = x \uparrow y$ from 117;$x \uparrow y$

4: $1 \cdot (x \uparrow y) = (x \uparrow y) \cdot 1$ from 105;$x \uparrow y$;1

Equality substitutions:

5: $\neg\, y + 0 = y \quad \vee \quad 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow (y)$

6: $\neg\, x \uparrow 0 = 1 \quad \vee \quad (x \uparrow y) \cdot (x \uparrow 0) = x \uparrow (y + 0) \quad \vee \quad \neg\, (x \uparrow y) \cdot (1) = x \uparrow (y + 0)$

7: $\neg\, 1 \cdot (x \uparrow y) = (x \uparrow y) \cdot 1 \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

Inferences:

8: $\neg\, x \uparrow 0 = 1 \quad \vee \quad \neg\, (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$ by
0: $\neg\, (x \uparrow y) \cdot (x \uparrow 0) = x \uparrow (y + 0)$
6: $\neg\, x \uparrow 0 = 1 \quad \vee \quad (x \uparrow y) \cdot (x \uparrow 0) = x \uparrow (y + 0) \quad \vee \quad \neg\, (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

9: $1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow y$ by
1: $y + 0 = y$
5: $\neg\, y + 0 = y \quad \vee \quad 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow y$

10: $\neg\, (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$ by
2: $x \uparrow 0 = 1$
8: $\neg\, x \uparrow 0 = 1 \quad \vee \quad \neg\, (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

11: $1 \cdot (x \uparrow y) = x \uparrow (y + 0)$ by

3: $1 \cdot (x \uparrow y) = x \uparrow y$

9: $1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow y$

12: $\neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$ by

4: $1 \cdot (x \uparrow y) = (x \uparrow y) \cdot 1$

7: $\neg\, 1 \cdot (x \uparrow y) = (x \uparrow y) \cdot 1 \quad \vee \quad \neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

13: $\neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0)$ by

10: $\neg\, (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

12: $\neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0) \quad \vee \quad (x \uparrow y) \cdot 1 = x \uparrow (y + 0)$

14: QEA by

11: $1 \cdot (x \uparrow y) = x \uparrow (y + 0)$

13: $\neg\, 1 \cdot (x \uparrow y) = x \uparrow (y + 0)$