Proof of Theorem 135

The theorem to be proved is

q is a power of two $\quad\rightarrow\quad 2\cdot q$ is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad$ [[(q) is a power of two] $\quad\&\quad$ [$\neg$ ($2\cdot q$) is a power of two]]

Special cases of the hypothesis and previous results:

0: $\quad q$ is a power of two $\quad$ from $\quad$ H:q

1: $\quad \neg\ 2\cdot q$ is a power of two $\quad$ from $\quad$ H:q

2: $\quad \neg\ q$ is a power of two $\quad\vee\quad 2\uparrow x = q$ $\quad$ from $\quad$ 129$^{\rightarrow}$;q:x

3: $\quad 2\cdot q$ is a power of two $\quad\vee\quad \neg\ \mathrm{S}x \leq 2\cdot q \quad\vee\quad \neg\ 2\uparrow(\mathrm{S}x) = 2\cdot q$ $\quad$ from $\quad$ 129$^{\leftarrow}$;$2\cdot q$;Sx

4: $\quad 2\cdot(2\uparrow x) = 2\uparrow(\mathrm{S}x)$ $\quad$ from $\quad$ 126;2;x

5: $\quad \mathrm{S}x \leq 2\uparrow(\mathrm{S}x)$ $\quad$ from $\quad$ 128;Sx

Equality substitutions:

6: $\quad \neg\ 2\uparrow x = q \quad\vee\quad \neg\ 2\cdot(2\uparrow x) = 2\uparrow(\mathrm{S}x) \quad\vee\quad 2\cdot(q) = 2\uparrow(\mathrm{S}x)$

7: $\quad \neg\ 2\uparrow(\mathrm{S}x) = 2\cdot q \quad\vee\quad \neg\ \mathrm{S}x \leq 2\uparrow(\mathrm{S}x) \quad\vee\quad \mathrm{S}x \leq 2\cdot q$

Inferences:

8: $\quad 2\uparrow x = q$ $\quad$ by

0: q is a power of two

2: $\neg\ q$ is a power of two $\quad\vee\quad 2\uparrow x = q$

9: $\quad \neg\ \mathrm{S}x \leq 2\cdot q \quad\vee\quad \neg\ 2\uparrow(\mathrm{S}x) = 2\cdot q$ $\quad$ by

1: $\neg\ 2\cdot q$ is a power of two

3: $2\cdot q$ is a power of two $\quad\vee\quad \neg\ \mathrm{S}x \leq 2\cdot q \quad\vee\quad \neg\ 2\uparrow(\mathrm{S}x) = 2\cdot q$

10: $\quad \neg\ 2\uparrow x = q \quad\vee\quad 2\uparrow(\mathrm{S}x) = 2\cdot q$ $\quad$ by

4: $2\cdot(2\uparrow x) = 2\uparrow(\mathrm{S}x)$

6: $\neg\ 2\uparrow x = q \quad\vee\quad \neg\ 2\cdot(2\uparrow x) = 2\uparrow(\mathrm{S}x) \quad\vee\quad 2\uparrow(\mathrm{S}x) = 2\cdot q$

11: $\neg\, 2 \uparrow (\mathrm{S}x) = 2 \cdot q \quad \vee \quad \mathrm{S}x \leq 2 \cdot q$ by

5: $\mathrm{S}x \leq 2 \uparrow (\mathrm{S}x)$

7: $\neg\, 2 \uparrow (\mathrm{S}x) = 2 \cdot q \quad \vee \quad \neg\, \mathrm{S}x \leq 2 \uparrow (\mathrm{S}x) \quad \vee \quad \mathrm{S}x \leq 2 \cdot q$

12: $2 \uparrow (\mathrm{S}x) = 2 \cdot q$ by

8: $2 \uparrow x = q$

10: $\neg\, 2 \uparrow x = q \quad \vee \quad 2 \uparrow (\mathrm{S}x) = 2 \cdot q$

13: $\neg\, \mathrm{S}x \leq 2 \cdot q$ by

12: $2 \uparrow (\mathrm{S}x) = 2 \cdot q$

9: $\neg\, \mathrm{S}x \leq 2 \cdot q \quad \vee \quad \neg\, 2 \uparrow (\mathrm{S}x) = 2 \cdot q$

14: $\mathrm{S}x \leq 2 \cdot q$ by

12: $2 \uparrow (\mathrm{S}x) = 2 \cdot q$

11: $\neg\, 2 \uparrow (\mathrm{S}x) = 2 \cdot q \quad \vee \quad \mathrm{S}x \leq 2 \cdot q$

15: QEA by

13: $\neg\, \mathrm{S}x \leq 2 \cdot q$

14: $\mathrm{S}x \leq 2 \cdot q$