Proof of Theorem 133i

The theorem to be proved is

$2 \uparrow x \neq 0 \quad \rightarrow \quad 2 \uparrow \mathrm{S}x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (2 \uparrow x) = (0)] \quad \& \quad [(2 \uparrow (\mathrm{S}x)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 2 \uparrow x = 0$ from H:x

1: $2 \uparrow (\mathrm{S}x) = 0$ from H:x

2: $\mathrm{S}(\mathrm{S}0) = 2$ from 116

3: $\mathrm{S}0 = 1$ from 115

4: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$ from 126;2;x

5: $\neg\, 2 \cdot (2 \uparrow x) = 0 \quad \vee \quad 2 = 0 \quad \vee \quad 2 \uparrow x = 0$ from 132;2;$2 \uparrow x$

6: $\neg\, \mathrm{S}1 = 0$ from 3;1

Equality substitutions:

7: $\neg\, 2 \uparrow (\mathrm{S}x) = 0 \quad \vee \quad \neg\, 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (2 \uparrow x) = 0$

8: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, \mathrm{S}(\mathrm{S}0) = 2 \quad \vee \quad \mathrm{S}(1) = 2$

9: $\neg\, 2 = 0 \quad \vee \quad \neg\, \mathrm{S}1 = 2 \quad \vee \quad \mathrm{S}1 = 0$

Inferences:

10: $\neg\, 2 \cdot (2 \uparrow x) = 0 \quad \vee \quad 2 = 0$ by
0: $\neg\, 2 \uparrow x = 0$
5: $\neg\, 2 \cdot (2 \uparrow x) = 0 \quad \vee \quad 2 = 0 \quad \vee \quad 2 \uparrow x = 0$

11: $\neg\, 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (2 \uparrow x) = 0$ by
1: $2 \uparrow (\mathrm{S}x) = 0$
7: $\neg\, 2 \uparrow (\mathrm{S}x) = 0 \quad \vee \quad \neg\, 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (2 \uparrow x) = 0$

12: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}1 = 2$ by
2: $\mathrm{S}(\mathrm{S}0) = 2$
8: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, \mathrm{S}(\mathrm{S}0) = 2 \quad \vee \quad \mathrm{S}1 = 2$

13: $\mathrm{S}1 = 2$ by

3: $\mathrm{S}0 = 1$

12: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}1 = 2$

14: $2 \cdot (2 \uparrow x) = 0$ by

4: $2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x)$

11: $\neg\, 2 \cdot (2 \uparrow x) = 2 \uparrow (\mathrm{S}x) \quad \vee \quad 2 \cdot (2 \uparrow x) = 0$

15: $\neg\, 2 = 0 \quad \vee \quad \neg\, \mathrm{S}1 = 2$ by

6: $\neg\, \mathrm{S}1 = 0$

9: $\neg\, 2 = 0 \quad \vee \quad \neg\, \mathrm{S}1 = 2 \quad \vee \quad \mathrm{S}1 = 0$

16: $\neg\, 2 = 0$ by

13: $\mathrm{S}1 = 2$

15: $\neg\, 2 = 0 \quad \vee \quad \neg\, \mathrm{S}1 = 2$

17: $2 = 0$ by

14: $2 \cdot (2 \uparrow x) = 0$

10: $\neg\, 2 \cdot (2 \uparrow x) = 0 \quad \vee \quad 2 = 0$

18: QEA by

16: $\neg\, 2 = 0$

17: $2 = 0$