Proof of Theorem 133b

The theorem to be proved is

$2 \uparrow 0 \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(2 \uparrow 0) = (0)]]$

Special cases of the hypothesis and previous results:

0: $2 \uparrow 0 = 0$ from H

1: $S0 = 1$ from 115

2: $2 \uparrow 0 = 1$ from 126;2

3: $\neg\, S1 = 0$ from 3;1

Equality substitutions:

4: $\neg\, 2 \uparrow 0 = 0 \quad \vee \quad \neg\, 2 \uparrow 0 = 1 \quad \vee \quad 0 = 1$

5: $\neg\, S0 = 1 \quad \vee \quad S0 = 0 \quad \vee \quad \neg\, 1 = 0$

6: $\neg\, 1 = 0 \quad \vee \quad S1 = 0 \quad \vee \quad \neg\, S0 = 0$

Inferences:

7: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 1 = 0$ by
0: $2 \uparrow 0 = 0$
4: $\neg\, 2 \uparrow 0 = 0 \quad \vee \quad \neg\, 2 \uparrow 0 = 1 \quad \vee \quad 1 = 0$

8: $S0 = 0 \quad \vee \quad \neg\, 1 = 0$ by
1: $S0 = 1$
5: $\neg\, S0 = 1 \quad \vee \quad S0 = 0 \quad \vee \quad \neg\, 1 = 0$

9: $1 = 0$ by
2: $2 \uparrow 0 = 1$
7: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 1 = 0$

10: $\neg\, 1 = 0 \quad \vee \quad \neg\, S0 = 0$ by
3: $\neg\, S1 = 0$
6: $\neg\, 1 = 0 \quad \vee \quad S1 = 0 \quad \vee \quad \neg\, S0 = 0$

11: $\mathrm{S}0 = 0$ by

9: $1 = 0$

8: $\mathrm{S}0 = 0 \quad \vee \quad \neg\, 1 = 0$

12: $\neg\, \mathrm{S}0 = 0$ by

9: $1 = 0$

10: $\neg\, 1 = 0 \quad \vee \quad \neg\, \mathrm{S}0 = 0$

13: QEA by

11: $\mathrm{S}0 = 0$

12: $\neg\, \mathrm{S}0 = 0$