Proof of Theorem 132

The theorem to be proved is

$x \cdot y = 0 \quad \rightarrow \quad x = 0 \quad \vee \quad y = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \cdot y) = (0)] \quad \& \quad [\neg\,(x) = (0)] \quad \& \quad [\neg\,(y) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x \cdot y = 0$ from H:x:y

1: $\neg\, 0 = x$ from H:x:y

2: $\neg\, 0 = y$ from H:x:y

3: $0 = y \quad \vee \quad \mathrm{S}(\mathrm{P}y) = y$ from 22;y

4: $x + (x \cdot (\mathrm{P}y)) = x \cdot (\mathrm{S}(\mathrm{P}y))$ from 99;x;Py

5: $\neg\, x + (x \cdot (\mathrm{P}y)) = 0 \quad \vee \quad 0 = x$ from 15;x;$x \cdot (\mathrm{P}y)$

Equality substitutions:

6: $\neg\, x \cdot y = 0 \quad \vee \quad \neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = 0$

7: $\neg\, \mathrm{S}(\mathrm{P}y) = y \quad \vee \quad \neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot (\mathrm{S}(\mathrm{P}y)) \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = x \cdot (y)$

Inferences:

8: $\neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = 0$ by
0: $x \cdot y = 0$
6: $\neg\, x \cdot y = 0 \quad \vee \quad \neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = 0$

9: $\neg\, x + (x \cdot (\mathrm{P}y)) = 0$ by
1: $\neg\, 0 = x$
5: $\neg\, x + (x \cdot (\mathrm{P}y)) = 0 \quad \vee \quad 0 = x$

10: $\mathrm{S}(\mathrm{P}y) = y$ by
2: $\neg\, 0 = y$
3: $0 = y \quad \vee \quad \mathrm{S}(\mathrm{P}y) = y$

11: $\neg\, \mathrm{S}(\mathrm{P}y) = y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = x \cdot y$ by

4: $x + (x \cdot (\mathrm{P}y)) = x \cdot (\mathrm{S}(\mathrm{P}y))$

7: $\neg\, \mathrm{S}(\mathrm{P}y) = y \quad \vee \quad \neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot (\mathrm{S}(\mathrm{P}y)) \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = x \cdot y$

12: $\neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y$ by

9: $\neg\, x + (x \cdot (\mathrm{P}y)) = 0$

8: $\neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = 0$

13: $x + (x \cdot (\mathrm{P}y)) = x \cdot y$ by

10: $\mathrm{S}(\mathrm{P}y) = y$

11: $\neg\, \mathrm{S}(\mathrm{P}y) = y \quad \vee \quad x + (x \cdot (\mathrm{P}y)) = x \cdot y$

14: QEA by

12: $\neg\, x + (x \cdot (\mathrm{P}y)) = x \cdot y$

13: $x + (x \cdot (\mathrm{P}y)) = x \cdot y$