Proof of Theorem 131

The theorem to be proved is

$2 \uparrow x$ is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[$\neg$ ($2 \uparrow x$) is a power of two]]

Special cases of the hypothesis and previous results:

0: $\neg\ 2 \uparrow x$ is a power of two from H:x

1: $2 \uparrow x$ is a power of two $\vee$ $\neg\ x \leq 2 \uparrow x$ $\vee$ $\neg\ 2 \uparrow x = 2 \uparrow x$ from 129$^{\Leftarrow}$;$2 \uparrow x$;x

2: $x \leq 2 \uparrow x$ from 128;x

3: $2 \uparrow x = 2 \uparrow x$ from 5;$2 \uparrow x$

Inferences:

4: $\neg\ x \leq 2 \uparrow x$ $\vee$ $\neg\ 2 \uparrow x = 2 \uparrow x$ by
0: $\neg\ 2 \uparrow x$ is a power of two
1: $2 \uparrow x$ is a power of two $\vee$ $\neg\ x \leq 2 \uparrow x$ $\vee$ $\neg\ 2 \uparrow x = 2 \uparrow x$

5: $\neg\ 2 \uparrow x = 2 \uparrow x$ by
2: $x \leq 2 \uparrow x$
4: $\neg\ x \leq 2 \uparrow x$ $\vee$ $\neg\ 2 \uparrow x = 2 \uparrow x$

6: QEA by
3: $2 \uparrow x = 2 \uparrow x$
5: $\neg\ 2 \uparrow x = 2 \uparrow x$