

## Proof of Theorem 130

The theorem to be proved is

1 is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H)  $[[\neg(1) \text{ is a power of two}]]$

### Special cases of the hypothesis and previous results:

0:  $\neg 1$  is a power of two from H

1: 1 is a power of two  $\vee \neg 0 \leq 1 \vee \neg 2 \uparrow 0 = 1$  from [129](#)<sup><</sup>;1;0

2:  $0 \leq 2 \uparrow 0$  from [128](#);0

3:  $2 \uparrow 0 = 1$  from [126](#);2

### Equality substitutions:

4:  $\neg 2 \uparrow 0 = 1 \vee \neg 0 \leq 2 \uparrow 0 \vee 0 \leq 1$

### Inferences:

5:  $\neg 0 \leq 1 \vee \neg 2 \uparrow 0 = 1$  by

0:  $\neg 1$  is a power of two

1: 1 is a power of two  $\vee \neg 0 \leq 1 \vee \neg 2 \uparrow 0 = 1$

6:  $\neg 2 \uparrow 0 = 1 \vee 0 \leq 1$  by

2:  $0 \leq 2 \uparrow 0$

4:  $\neg 2 \uparrow 0 = 1 \vee \neg 0 \leq 2 \uparrow 0 \vee 0 \leq 1$

7:  $\neg 0 \leq 1$  by

3:  $2 \uparrow 0 = 1$

5:  $\neg 0 \leq 1 \vee \neg 2 \uparrow 0 = 1$

8:  $0 \leq 1$  by

3:  $2 \uparrow 0 = 1$

6:  $\neg 2 \uparrow 0 = 1 \vee 0 \leq 1$

9: *QEA* by

7:  $\neg 0 \leq 1$

8:  $0 \leq 1$