Proof of Theorem 128

The theorem to be proved is

$x \leq 2 \uparrow x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg (x) \leq (2 \uparrow x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \leq 2 \uparrow x$ from H:x

1: $x < 2 \uparrow x$ from 127;x

2: $\neg\, x < 2 \uparrow x \quad \vee \quad x \leq 2 \uparrow x$ from $56^{\rightarrow}$;x;$2 \uparrow x$

Inferences:

3: $\neg\, x < 2 \uparrow x$ by
0: $\neg\, x \leq 2 \uparrow x$
2: $\neg\, x < 2 \uparrow x \quad \vee \quad x \leq 2 \uparrow x$

4: QEA by
1: $x < 2 \uparrow x$
3: $\neg\, x < 2 \uparrow x$