Proof of Theorem 127b

The theorem to be proved is

$0 < 2 \uparrow 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(0) < (2 \uparrow 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 < 2 \uparrow 0$ from H

1: $S0 = 1$ from 115

2: $2 \uparrow 0 = 1$ from 126;2

3: $0 < S0$ from 125;0

Equality substitutions:

4: $\neg\, S0 = 1 \quad \vee \quad \neg\, 0 < S0 \quad \vee \quad 0 < 1$

5: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 0 < 2 \uparrow 0 \quad \vee \quad \neg\, 0 < 1$

Inferences:

6: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 0 < 1$ by
- 0: $\neg\, 0 < 2 \uparrow 0$
- 5: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad 0 < 2 \uparrow 0 \quad \vee \quad \neg\, 0 < 1$

7: $\neg\, 0 < S0 \quad \vee \quad 0 < 1$ by
- 1: $S0 = 1$
- 4: $\neg\, S0 = 1 \quad \vee \quad \neg\, 0 < S0 \quad \vee \quad 0 < 1$

8: $\neg\, 0 < 1$ by
- 2: $2 \uparrow 0 = 1$
- 6: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 0 < 1$

9: $0 < 1$ by
- 3: $0 < S0$
- 7: $\neg\, 0 < S0 \quad \vee \quad 0 < 1$

10: QEA by
- 8: $\neg\, 0 < 1$
- 9: $0 < 1$