Proof of Theorem 125

The theorem to be proved is

$x < \mathrm{S}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x) < (\mathrm{S}x)]]$

Special cases of the hypothesis and previous results:

0:   $\neg\, x < \mathrm{S}x$ from   H:x

1:   $\neg\, \mathrm{S}x \leq x$ from   <u>59</u>;x

2:   $\mathrm{S}x \leq x \;\; \vee \;\; x < \mathrm{S}x$ from   <u>79</u>;$\mathrm{S}x$;x

Inferences:

3:   $\mathrm{S}x \leq x$ by
   0: $\neg\, x < \mathrm{S}x$
   2: $\mathrm{S}x \leq x \;\; \vee \;\; x < \mathrm{S}x$

4:   QEA by
   1: $\neg\, \mathrm{S}x \leq x$
   3: $\mathrm{S}x \leq x$

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