Proof of Theorem 123

The theorem to be proved is

$x < y \quad \rightarrow \quad y \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) < (y)] \quad \& \quad [(y) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x:y

1: $0 = y$ from H:x:y

2: $\neg\, x < y \quad \vee \quad x \leq y$ from 56$^{\rightarrow}$;x;y

3: $\neg\, x < y \quad \vee \quad \neg\, y = x$ from 56$^{\rightarrow}$;x;y

4: $\neg\, x \leq 0 \quad \vee \quad 0 = x$ from 57;x

Equality substitutions:

5: $\neg\, 0 = y \quad \vee \quad x \leq 0 \quad \vee \quad \neg\, x \leq y$

6: $\neg\, 0 = y \quad \vee \quad \neg\, 0 = x \quad \vee \quad y = x$

Inferences:

7: $x \leq y$ by
0: $x < y$
2: $\neg\, x < y \quad \vee \quad x \leq y$

8: $\neg\, y = x$ by
0: $x < y$
3: $\neg\, x < y \quad \vee \quad \neg\, y = x$

9: $x \leq 0 \quad \vee \quad \neg\, x \leq y$ by
1: $0 = y$
5: $\neg\, 0 = y \quad \vee \quad x \leq 0 \quad \vee \quad \neg\, x \leq y$

10: $\neg\, 0 = x \quad \vee \quad y = x$ by
1: $0 = y$
6: $\neg\, 0 = y \quad \vee \quad \neg\, 0 = x \quad \vee \quad y = x$

11: $x \leq 0$ by

7: $x \leq y$

9: $x \leq 0 \quad \vee \quad \neg\, x \leq y$

12: $\neg\, 0 = x$ by

8: $\neg\, y = x$

10: $\neg\, 0 = x \quad \vee \quad y = x$

13: $0 = x$ by

11: $x \leq 0$

4: $\neg\, x \leq 0 \quad \vee \quad 0 = x$

14: QEA by

12: $\neg\, 0 = x$

13: $0 = x$