Proof of Theorem 120

The theorem to be proved is

$x + y = x + z \quad \rightarrow \quad y = z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x + y) = (x + z)] \quad \& \quad [\neg\, (y) = (z)]]$

Special cases of the hypothesis and previous results:

0: $x + z = x + y$ from H:x:y:z

1: $\neg\, z = y$ from H:x:y:z

2: $y + x = x + y$ from 98;x;y

3: $z + x = x + z$ from 98;x;z

4: $\neg\, z + x = y + x \quad \vee \quad z = y$ from 119;y;x;z

Equality substitutions:

5: $\neg\, x + z = x + y \quad \vee \quad y + x = x + z \quad \vee \quad \neg\, y + x = x + y$

6: $\neg\, z + x = x + z \quad \vee \quad z + x = y + x \quad \vee \quad \neg\, x + z = y + x$

Inferences:

7: $y + x = x + z \quad \vee \quad \neg\, y + x = x + y$ by
0: $x + z = x + y$
5: $\neg\, x + z = x + y \quad \vee \quad y + x = x + z \quad \vee \quad \neg\, y + x = x + y$

8: $\neg\, z + x = y + x$ by
1: $\neg\, z = y$
4: $\neg\, z + x = y + x \quad \vee \quad z = y$

9: $y + x = x + z$ by
2: $y + x = x + y$
7: $y + x = x + z \quad \vee \quad \neg\, y + x = x + y$

10: $z + x = y + x \quad \vee \quad \neg\, y + x = x + z$ by
3: $z + x = x + z$
6: $\neg\, z + x = x + z \quad \vee \quad z + x = y + x \quad \vee \quad \neg\, y + x = x + z$

11: $\neg\, y + x = x + z$ by

8: $\neg\, z + x = y + x$

10: $z + x = y + x \quad \vee \quad \neg\, y + x = x + z$

12: QEA by

9: $y + x = x + z$

11: $\neg\, y + x = x + z$