Proof of Theorem 119b

The theorem to be proved is

$y + 0 = z + 0 \quad \rightarrow \quad y = z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(y + 0) = (z + 0)] \quad \& \quad [\neg\, (y) = (z)]]$

Special cases of the hypothesis and previous results:

0: $z + 0 = y + 0$ from H:y:z

1: $\neg\, z = y$ from H:y:z

2: $y + 0 = y$ from 12;y

3: $z + 0 = z$ from 12;z

Equality substitutions:

4: $\neg\, z + 0 = y + 0 \quad \vee \quad z + 0 = y \quad \vee \quad \neg\, y + 0 = y$

5: $\neg\, z + 0 = z \quad \vee \quad \neg\, z + 0 = y \quad \vee \quad z = y$

Inferences:

6: $z + 0 = y \quad \vee \quad \neg\, y + 0 = y$ by
0: $z + 0 = y + 0$
4: $\neg\, z + 0 = y + 0 \quad \vee \quad z + 0 = y \quad \vee \quad \neg\, y + 0 = y$

7: $\neg\, z + 0 = z \quad \vee \quad \neg\, z + 0 = y$ by
1: $\neg\, z = y$
5: $\neg\, z + 0 = z \quad \vee \quad \neg\, z + 0 = y \quad \vee \quad z = y$

8: $z + 0 = y$ by
2: $y + 0 = y$
6: $z + 0 = y \quad \vee \quad \neg\, y + 0 = y$

9: $\neg\, z + 0 = y$ by
3: $z + 0 = z$
7: $\neg\, z + 0 = z \quad \vee \quad \neg\, z + 0 = y$

10: QEA by
8: $z + 0 = y$
9: $\neg\, z + 0 = y$