Proof of Theorem 118

The theorem to be proved is

$2 \cdot x = x + x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (2 \cdot x) = (x + x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ 2 \cdot x = x + x$ from H:x

1: $\mathrm{S(S0)} = 2$ from 116

2: $\mathrm{S0} = 1$ from 115

3: $(1 \cdot x) + x = (\mathrm{S1}) \cdot x$ from 104;1;x

4: $1 \cdot x = x$ from 117;x

Equality substitutions:

5: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{S(S0)} = 2 \quad \vee \quad \mathrm{S(1)} = 2$

6: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ (1 \cdot x) + x = 2 \cdot x \quad \vee \quad (x) + x = 2 \cdot x$

7: $\neg\ \mathrm{S1} = 2 \quad \vee \quad \neg\ (1 \cdot x) + x = (\mathrm{S1}) \cdot x \quad \vee \quad (1 \cdot x) + x = (2) \cdot x$

Inferences:

8: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ (1 \cdot x) + x = 2 \cdot x$ by
0: $\neg\ 2 \cdot x = x + x$
6: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ (1 \cdot x) + x = 2 \cdot x \quad \vee \quad 2 \cdot x = x + x$

9: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{S1} = 2$ by
1: $\mathrm{S(S0)} = 2$
5: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{S(S0)} = 2 \quad \vee \quad \mathrm{S1} = 2$

10: $\mathrm{S1} = 2$ by
2: $\mathrm{S0} = 1$
9: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{S1} = 2$

11: $\neg\,\mathrm{S}1 = 2 \quad \vee \quad (1\cdot x) + x = 2\cdot x$ by

3: $(1\cdot x) + x = (\mathrm{S}1)\cdot x$

7: $\neg\,\mathrm{S}1 = 2 \quad \vee \quad \neg\,(1\cdot x) + x = (\mathrm{S}1)\cdot x \quad \vee \quad (1\cdot x) + x = 2\cdot x$

12: $\neg\,(1\cdot x) + x = 2\cdot x$ by

4: $1\cdot x = x$

8: $\neg\,1\cdot x = x \quad \vee \quad \neg\,(1\cdot x) + x = 2\cdot x$

13: $(1\cdot x) + x = 2\cdot x$ by

10: $\mathrm{S}1 = 2$

11: $\neg\,\mathrm{S}1 = 2 \quad \vee \quad (1\cdot x) + x = 2\cdot x$

14: QEA by

12: $\neg\,(1\cdot x) + x = 2\cdot x$

13: $(1\cdot x) + x = 2\cdot x$