Proof of Theorem 117

The theorem to be proved is

$1 \cdot x = x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(1 \cdot x) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 1 \cdot x = x$ from H:x

1: $\mathrm{S}0 = 1$ from 115

2: $(0 \cdot x) + x = (\mathrm{S}0) \cdot x$ from 104;0;x

3: $0 + x = x$ from 97;x

4: $0 \cdot x = 0$ from 103;x

Equality substitutions:

5: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\,(0 \cdot x) + x = (\mathrm{S}0) \cdot x \quad \vee \quad (0 \cdot x) + x = (1) \cdot x$

6: $\neg\, 0 + x = x \quad \vee \quad \neg\, 1 \cdot x = 0 + x \quad \vee \quad 1 \cdot x = x$

7: $\neg\, 0 \cdot x = 0 \quad \vee \quad \neg\,(0 \cdot x) + x = 1 \cdot x \quad \vee \quad (0) + x = 1 \cdot x$

Inferences:

8: $\neg\, 0 + x = x \quad \vee \quad \neg\, 1 \cdot x = 0 + x$ by
0: $\neg\, 1 \cdot x = x$
6: $\neg\, 0 + x = x \quad \vee \quad \neg\, 1 \cdot x = 0 + x \quad \vee \quad 1 \cdot x = x$

9: $\neg\,(0 \cdot x) + x = (\mathrm{S}0) \cdot x \quad \vee \quad (0 \cdot x) + x = 1 \cdot x$ by
1: $\mathrm{S}0 = 1$
5: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\,(0 \cdot x) + x = (\mathrm{S}0) \cdot x \quad \vee \quad (0 \cdot x) + x = 1 \cdot x$

10: $(0 \cdot x) + x = 1 \cdot x$ by
2: $(0 \cdot x) + x = (\mathrm{S}0) \cdot x$
9: $\neg\,(0 \cdot x) + x = (\mathrm{S}0) \cdot x \quad \vee \quad (0 \cdot x) + x = 1 \cdot x$

11: $\neg\, 1 \cdot x = 0 + x$ by

3: $0 + x = x$

8: $\neg\, 0 + x = x \quad \vee \quad \neg\, 1 \cdot x = 0 + x$

12: $\neg\, (0 \cdot x) + x = 1 \cdot x \quad \vee \quad 1 \cdot x = 0 + x$ by

4: $0 \cdot x = 0$

7: $\neg\, 0 \cdot x = 0 \quad \vee \quad \neg\, (0 \cdot x) + x = 1 \cdot x \quad \vee \quad 1 \cdot x = 0 + x$

13: $1 \cdot x = 0 + x$ by

10: $(0 \cdot x) + x = 1 \cdot x$

12: $\neg\, (0 \cdot x) + x = 1 \cdot x \quad \vee \quad 1 \cdot x = 0 + x$

14: QEA by

11: $\neg\, 1 \cdot x = 0 + x$

13: $1 \cdot x = 0 + x$