

## Proof of Theorem 111

The theorem to be proved is

$$\neg [x < y \ \& \ y < Sx]$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[(x) < (y)] \ \& \ [(y) < (Sx)]]$$

### Special cases of the hypothesis and previous results:

$$0: \ x < y \quad \text{from } H:x:y$$

$$1: \ y < Sx \quad \text{from } H:x:y$$

$$2: \ \neg y < Sx \ \vee \ y \leq x \quad \text{from } \underline{109};y;x$$

$$3: \ \neg x < y \ \vee \ \neg y \leq x \quad \text{from } \underline{78};x;y$$

### Inferences:

$$4: \ \neg y \leq x \quad \text{by}$$

$$0: \ x < y$$

$$3: \ \neg x < y \ \vee \ \neg y \leq x$$

$$5: \ y \leq x \quad \text{by}$$

$$1: \ y < Sx$$

$$2: \ \neg y < Sx \ \vee \ y \leq x$$

$$6: \ QEA \quad \text{by}$$

$$4: \ \neg y \leq x$$

$$5: \ y \leq x$$