Proof of Theorem 108

The theorem to be proved is

$x < y \quad \rightarrow \quad x + (y - x) = y \quad \& \quad y - x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) < (y)] \quad \& \quad [\neg\, (x + (y - x)) = (y) \quad \vee \quad (y - x) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x:y

1: $\neg\, x + (y - x) = y \quad \vee \quad y - x = 0$ from H:x:y

2: $\neg\, x \leq y \quad \vee \quad x + (y - x) = y$ from 68;x;y

3: $\neg\, x < y \quad \vee \quad x \leq y$ from $56^{\rightarrow}$;x;y

4: $\neg\, x < y \quad \vee \quad \neg\, y = x$ from $56^{\rightarrow}$;x;y

5: $x + 0 = x$ from 12;x

Equality substitutions:

6: $\neg\, y - x = 0 \quad \vee \quad \neg\, x + (y - x) = y \quad \vee \quad x + (0) = y$

7: $\neg\, x + 0 = x \quad \vee \quad \neg\, x + 0 = y \quad \vee \quad x = y$

Inferences:

8: $x \leq y$ by
0: $x < y$
3: $\neg\, x < y \quad \vee \quad x \leq y$

9: $\neg\, y = x$ by
0: $x < y$
4: $\neg\, x < y \quad \vee \quad \neg\, y = x$

10: $\neg\, x + 0 = y \quad \vee \quad y = x$ by
5: $x + 0 = x$
7: $\neg\, x + 0 = x \quad \vee \quad \neg\, x + 0 = y \quad \vee \quad y = x$

11: $x + (y - x) = y$ by

8: $x \leq y$

2: $\neg\, x \leq y \quad \vee \quad x + (y - x) = y$

12: $\neg\, x + 0 = y$ by

9: $\neg\, y = x$

10: $\neg\, x + 0 = y \quad \vee \quad y = x$

13: $y - x = 0$ by

11: $x + (y - x) = y$

1: $\neg\, x + (y - x) = y \quad \vee \quad y - x = 0$

14: $\neg\, y - x = 0 \quad \vee \quad x + 0 = y$ by

11: $x + (y - x) = y$

6: $\neg\, y - x = 0 \quad \vee \quad \neg\, x + (y - x) = y \quad \vee \quad x + 0 = y$

15: $\neg\, y - x = 0$ by

12: $\neg\, x + 0 = y$

14: $\neg\, y - x = 0 \quad \vee \quad x + 0 = y$

16: QEA by

13: $y - x = 0$

15: $\neg\, y - x = 0$