Proof of Theorem 105b

The theorem to be proved is

$x \cdot 0 = 0 \cdot x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x \cdot 0) = (0 \cdot x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, 0 \cdot x = x \cdot 0$ from H:x

1: $x \cdot 0 = 0$ from 100;x

2: $0 \cdot x = 0$ from 103;x

Equality substitutions:

3: $\neg\, x \cdot 0 = 0 \quad \vee \quad 0 \cdot x = x \cdot 0 \quad \vee \quad \neg\, 0 \cdot x = 0$

Inferences:

4: $\neg\, x \cdot 0 = 0 \quad \vee \quad \neg\, 0 \cdot x = 0$ by

0: $\neg\, 0 \cdot x = x \cdot 0$

3: $\neg\, x \cdot 0 = 0 \quad \vee \quad 0 \cdot x = x \cdot 0 \quad \vee \quad \neg\, 0 \cdot x = 0$

5: $\neg\, 0 \cdot x = 0$ by

1: $x \cdot 0 = 0$

4: $\neg\, x \cdot 0 = 0 \quad \vee \quad \neg\, 0 \cdot x = 0$

6: QEA by

2: $0 \cdot x = 0$

5: $\neg\, 0 \cdot x = 0$