Proof of Theorem 104b

The theorem to be proved is

$\mathrm{S}x \cdot 0 = x \cdot 0 + 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, ((\mathrm{S}x) \cdot 0) = ((x \cdot 0) + 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (x \cdot 0) + 0 = (\mathrm{S}x) \cdot 0$ from H:x

1: $(\mathrm{S}x) \cdot 0 = 0$ from 100;Sx

2: $x \cdot 0 = 0$ from 100;x

3: $0 + 0 = 0$ from 12;0

Equality substitutions:

4: $\neg\, (\mathrm{S}x) \cdot 0 = 0 \quad \vee \quad (x \cdot 0) + 0 = (\mathrm{S}x) \cdot 0 \quad \vee \quad \neg\, (x \cdot 0) + 0 = 0$

5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot 0) + 0 = 0 \quad \vee \quad \neg\, (0) + 0 = 0$

Inferences:

6: $\neg\, (\mathrm{S}x) \cdot 0 = 0 \quad \vee \quad \neg\, (x \cdot 0) + 0 = 0$ by

0: $\neg\, (x \cdot 0) + 0 = (\mathrm{S}x) \cdot 0$

4: $\neg\, (\mathrm{S}x) \cdot 0 = 0 \quad \vee \quad (x \cdot 0) + 0 = (\mathrm{S}x) \cdot 0 \quad \vee \quad \neg\, (x \cdot 0) + 0 = 0$

7: $\neg\, (x \cdot 0) + 0 = 0$ by

1: $(\mathrm{S}x) \cdot 0 = 0$

6: $\neg\, (\mathrm{S}x) \cdot 0 = 0 \quad \vee \quad \neg\, (x \cdot 0) + 0 = 0$

8: $(x \cdot 0) + 0 = 0 \quad \vee \quad \neg\, 0 + 0 = 0$ by

2: $x \cdot 0 = 0$

5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot 0) + 0 = 0 \quad \vee \quad \neg\, 0 + 0 = 0$

9: $(x \cdot 0) + 0 = 0$ by

3: $0 + 0 = 0$

8: $(x \cdot 0) + 0 = 0 \quad \vee \quad \neg\, 0 + 0 = 0$

10: QEA by

7: $\neg\, (x \cdot 0) + 0 = 0$

9: $(x \cdot 0) + 0 = 0$