Proof of Theorem 102b

The theorem to be proved is

$x \cdot (y \cdot 0) = (x \cdot y) \cdot 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x \cdot (y \cdot 0)) = ((x \cdot y) \cdot 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \cdot (y \cdot 0) = (x \cdot y) \cdot 0$ from H:x:y

1: $y \cdot 0 = 0$ from 100;y

2: $x \cdot 0 = 0$ from 100;x

3: $(x \cdot y) \cdot 0 = 0$ from 100;$x \cdot y$

Equality substitutions:

4: $\neg\, y \cdot 0 = 0 \quad \vee \quad x \cdot (y \cdot 0) = (x \cdot y) \cdot 0 \quad \vee \quad \neg\, x \cdot (0) = (x \cdot y) \cdot 0$

5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot y) \cdot 0 = x \cdot 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = 0$

Inferences:

6: $\neg\, y \cdot 0 = 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = x \cdot 0$ by

0: $\neg\, x \cdot (y \cdot 0) = (x \cdot y) \cdot 0$

4: $\neg\, y \cdot 0 = 0 \quad \vee \quad x \cdot (y \cdot 0) = (x \cdot y) \cdot 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = x \cdot 0$

7: $\neg\, (x \cdot y) \cdot 0 = x \cdot 0$ by

1: $y \cdot 0 = 0$

6: $\neg\, y \cdot 0 = 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = x \cdot 0$

8: $(x \cdot y) \cdot 0 = x \cdot 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = 0$ by

2: $x \cdot 0 = 0$

5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot y) \cdot 0 = x \cdot 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = 0$

9: $(x \cdot y) \cdot 0 = x \cdot 0$ by

3: $(x \cdot y) \cdot 0 = 0$

8: $(x \cdot y) \cdot 0 = x \cdot 0 \quad \vee \quad \neg\, (x \cdot y) \cdot 0 = 0$

10: QEA by

7: $\neg\, (x \cdot y) \cdot 0 = x \cdot 0$

9: $(x \cdot y) \cdot 0 = x \cdot 0$