Proof of Theorem 101b

The theorem to be proved is

$x \cdot (y + 0) = x \cdot y + x \cdot 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x \cdot (y + 0)) = ((x \cdot y) + (x \cdot 0))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (x \cdot y) + (x \cdot 0) = x \cdot (y + 0)$ from H:x:y

1: $y + 0 = y$ from 12;y

2: $x \cdot 0 = 0$ from 100;x

3: $(x \cdot y) + 0 = x \cdot y$ from 12;$x \cdot y$

Equality substitutions:

4: $\neg\, y + 0 = y \quad \vee \quad (x \cdot y) + (x \cdot 0) = x \cdot (y + 0) \quad \vee \quad \neg\, (x \cdot y) + (x \cdot 0) = x \cdot (y)$

5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot y) + (x \cdot 0) = x \cdot y \quad \vee \quad \neg\, (x \cdot y) + (0) = x \cdot y$

Inferences:

6: $\neg\, y + 0 = y \quad \vee \quad \neg\, (x \cdot y) + (x \cdot 0) = x \cdot y$ by
0: $\neg\, (x \cdot y) + (x \cdot 0) = x \cdot (y + 0)$
4: $\neg\, y + 0 = y \quad \vee \quad (x \cdot y) + (x \cdot 0) = x \cdot (y + 0) \quad \vee \quad \neg\, (x \cdot y) + (x \cdot 0) = x \cdot y$

7: $\neg\, (x \cdot y) + (x \cdot 0) = x \cdot y$ by
1: $y + 0 = y$
6: $\neg\, y + 0 = y \quad \vee \quad \neg\, (x \cdot y) + (x \cdot 0) = x \cdot y$

8: $(x \cdot y) + (x \cdot 0) = x \cdot y \quad \vee \quad \neg\, (x \cdot y) + 0 = x \cdot y$ by
2: $x \cdot 0 = 0$
5: $\neg\, x \cdot 0 = 0 \quad \vee \quad (x \cdot y) + (x \cdot 0) = x \cdot y \quad \vee \quad \neg\, (x \cdot y) + 0 = x \cdot y$

9: $(x \cdot y) + (x \cdot 0) = x \cdot y$ by
3: $(x \cdot y) + 0 = x \cdot y$
8: $(x \cdot y) + (x \cdot 0) = x \cdot y \quad \vee \quad \neg\, (x \cdot y) + 0 = x \cdot y$

10: QEA by
7: $\neg\, (x \cdot y) + (x \cdot 0) = x \cdot y$
9: $(x \cdot y) + (x \cdot 0) = x \cdot y$