

## Proof of Theorem 100

The theorem to be proved is

$$x \cdot 0 = 0 \quad \& \quad x \cdot Sy = x \cdot y + x$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[\neg (x \cdot 0) = 0) \quad \vee \quad \neg (x \cdot (Sy)) = ((x \cdot y) + x)]]$$

### Special cases of the hypothesis and previous results:

- 0:  $\neg x \cdot 0 = 0 \quad \vee \quad \neg (x \cdot y) + x = x \cdot (Sy)$  from  $H; x; y$
- 1:  $x \cdot 0 = 0$  from [99](#);  $x; y$
- 2:  $x + (x \cdot y) = x \cdot (Sy)$  from [99](#);  $x; y$
- 3:  $x + (x \cdot y) = (x \cdot y) + x$  from [98](#);  $x; x \cdot y$

### Equality substitutions:

$$4: \quad \neg x + (x \cdot y) = x \cdot (Sy) \quad \vee \quad \neg \textcolor{red}{x} + \textcolor{red}{(x \cdot y)} = (x \cdot y) + x \quad \vee \quad \textcolor{red}{x} \cdot \textcolor{red}{(Sy)} = (x \cdot y) + x$$

### Inferences:

- 5:  $\neg (x \cdot y) + x = x \cdot (Sy)$  by
  - 1:  $\textcolor{red}{x} \cdot \textcolor{red}{0} = 0$
  - 0:  $\neg \textcolor{red}{x} \cdot \textcolor{red}{0} = 0 \quad \vee \quad \neg (x \cdot y) + x = x \cdot (Sy)$
- 6:  $\neg x + (x \cdot y) = (x \cdot y) + x \quad \vee \quad (x \cdot y) + x = x \cdot (Sy)$  by
  - 2:  $\textcolor{red}{x} + \textcolor{red}{(x \cdot y)} = \textcolor{red}{x} \cdot \textcolor{red}{(Sy)}$
  - 4:  $\neg \textcolor{red}{x} + \textcolor{red}{(x \cdot y)} = \textcolor{red}{x} \cdot \textcolor{red}{(Sy)} \quad \vee \quad \neg x + (x \cdot y) = (x \cdot y) + x \quad \vee \quad (x \cdot y) + x = x \cdot (Sy)$
- 7:  $(x \cdot y) + x = x \cdot (Sy)$  by
  - 3:  $\textcolor{red}{x} + \textcolor{red}{(x \cdot y)} = \textcolor{red}{(x \cdot y)} + x$
  - 6:  $\neg \textcolor{red}{x} + \textcolor{red}{(x \cdot y)} = \textcolor{red}{(x \cdot y)} + x \quad \vee \quad (x \cdot y) + x = x \cdot (Sy)$
- 8:  $QEA$  by
  - 5:  $\neg (x \cdot y) + x = x \cdot (Sy)$
  - 7:  $(x \cdot y) + x = x \cdot (Sy)$