Proof of Theorem 013

The theorem to be proved is

last-bit(x, b_1) & last-bit(x, b_2) $\rightarrow$ $b_1 = b_2$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[last-bit$((xb_1))$] & [last-bit$((xb_2))$] & [$\neg$ $(b_1) = (b_2)$]]

Special cases of the hypothesis and previous results:

0: last-bit$((xb_1))$ from H:x:b_1:b_2

1: last-bit$((xb_2))$ from H:x:b_1:b_2

2: $\neg$ $b_2 = b_1$ from H:x:b_1:b_2

3: $\neg$ last-bit$((xb_1))$ $\vee$ $\epsilon = b_1$ from 012$^{\rightarrow}$;x;b_1

4: $\neg$ last-bit$((xb_2))$ $\vee$ $\epsilon = b_2$ from 012$^{\rightarrow}$;x;b_2

Equality substitutions:

5: $\neg$ $\epsilon = b_1$ $\vee$ $\neg$ $\epsilon = b_2$ $\vee$ $b_1 = b_2$

Inferences:

6: $\epsilon = b_1$ by
0: last-bit$((xb_1))$
3: $\neg$ last-bit$((xb_1))$ $\vee$ $\epsilon = b_1$

7: $\epsilon = b_2$ by
1: last-bit$((xb_2))$
4: $\neg$ last-bit$((xb_2))$ $\vee$ $\epsilon = b_2$

8: $\neg$ $\epsilon = b_1$ $\vee$ $\neg$ $\epsilon = b_2$ by
2: $\neg$ $b_2 = b_1$
5: $\neg$ $\epsilon = b_1$ $\vee$ $\neg$ $\epsilon = b_2$ $\vee$ $b_2 = b_1$

9: $\neg$ $\epsilon = b_2$ by
6: $\epsilon = b_1$
8: $\neg$ $\epsilon = b_1$ $\vee$ $\neg$ $\epsilon = b_2$

10: QEA by
7: $\epsilon = b_2$
9: $\neg$ $\epsilon = b_2$