Proof of Theorem 011

The theorem to be proved is

x ends with y & y ends with x $\rightarrow$ $x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x)$ ends with $(y)]$ & $[(y)$ ends with $(x)]$ & $[\neg\,(x) = (y)]]$

Special cases of the hypothesis and previous results:

0: x ends with y from H:x:y

1: y ends with x from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $\neg\, x$ ends with y $\vee$ $c \oplus y = x$ from $288^{\rightarrow}$;x;y:c

4: $\neg\, y$ ends with x $\vee$ $d \oplus x = y$ from $288^{\rightarrow}$;y;x:d

5: $\epsilon \oplus x = x$ from 194;x

6: $c \oplus (d \oplus x) = (c \oplus d) \oplus x$ from 183;c;d;x

7: $\neg\,(c \oplus d) \oplus x = \epsilon \oplus x$ $\vee$ $c \oplus d = \epsilon$ from 197;ϵ;x;$c \oplus d$

8: $\neg\, c \oplus d = \epsilon$ $\vee$ $\epsilon = d$ from 204;c;d

Equality substitutions:

9: $\neg\, c \oplus y = x$ $\vee$ $\epsilon \oplus x = c \oplus y$ $\vee$ $\neg\, \epsilon \oplus x = x$

10: $\neg\, d \oplus x = y$ $\vee$ $\neg\, c \oplus (d \oplus x) = (c \oplus d) \oplus x$ $\vee$ $c \oplus (y) = (c \oplus d) \oplus x$

11: $\neg\, \epsilon \oplus x = x$ $\vee$ $\neg\, \epsilon \oplus x = y$ $\vee$ $x = y$

12: $\neg\, \epsilon = d$ $\vee$ $\epsilon \oplus x = y$ $\vee$ $\neg\, d \oplus x = y$

13: $\neg\,(c \oplus d) \oplus x = c \oplus y$ $\vee$ $(c \oplus d) \oplus x = \epsilon \oplus x$ $\vee$ $\neg\, c \oplus y = \epsilon \oplus x$

Inferences:

14: $c \oplus y = x$ by

0: x ends with y

3: $\neg\, x$ ends with y $\vee$ $c \oplus y = x$

15: $d \oplus x = y$ by

1: y ends with x

4: $\neg\, y$ ends with $x \quad \vee \quad d \oplus x = y$

16: $\neg\, \epsilon \oplus x = x \quad \vee \quad \neg\, \epsilon \oplus x = y$ by

2: $\neg\, y = x$

11: $\neg\, \epsilon \oplus x = x \quad \vee \quad \neg\, \epsilon \oplus x = y \quad \vee \quad y = x$

17: $\neg\, c \oplus y = x \quad \vee \quad \epsilon \oplus x = c \oplus y$ by

5: $\epsilon \oplus x = x$

9: $\neg\, c \oplus y = x \quad \vee \quad \epsilon \oplus x = c \oplus y \quad \vee \quad \neg\, \epsilon \oplus x = x$

18: $\neg\, \epsilon \oplus x = y$ by

5: $\epsilon \oplus x = x$

16: $\neg\, \epsilon \oplus x = x \quad \vee \quad \neg\, \epsilon \oplus x = y$

19: $\neg\, d \oplus x = y \quad \vee \quad (c \oplus d) \oplus x = c \oplus y$ by

6: $c \oplus (d \oplus x) = (c \oplus d) \oplus x$

10: $\neg\, d \oplus x = y \quad \vee \quad \neg\, c \oplus (d \oplus x) = (c \oplus d) \oplus x \quad \vee \quad (c \oplus d) \oplus x = c \oplus y$

20: $\epsilon \oplus x = c \oplus y$ by

14: $c \oplus y = x$

17: $\neg\, c \oplus y = x \quad \vee \quad \epsilon \oplus x = c \oplus y$

21: $\neg\, \epsilon = d \quad \vee \quad \epsilon \oplus x = y$ by

15: $d \oplus x = y$

12: $\neg\, \epsilon = d \quad \vee \quad \epsilon \oplus x = y \quad \vee \quad \neg\, d \oplus x = y$

22: $(c \oplus d) \oplus x = c \oplus y$ by

15: $d \oplus x = y$

19: $\neg\, d \oplus x = y \quad \vee \quad (c \oplus d) \oplus x = c \oplus y$

23: $\neg\, \epsilon = d$ by

18: $\neg\, \epsilon \oplus x = y$

21: $\neg\, \epsilon = d \quad \vee \quad \epsilon \oplus x = y$

24: $\neg\, (c \oplus d) \oplus x = c \oplus y \quad \vee \quad (c \oplus d) \oplus x = \epsilon \oplus x$ by

20: $\epsilon \oplus x = c \oplus y$

13: $\neg\, (c \oplus d) \oplus x = c \oplus y \quad \vee \quad (c \oplus d) \oplus x = \epsilon \oplus x \quad \vee \quad \neg\, \epsilon \oplus x = c \oplus y$

25: $(c \oplus d) \oplus x = \epsilon \oplus x$ by

22: $(c \oplus d) \oplus x = c \oplus y$

24: $\neg\, (c \oplus d) \oplus x = c \oplus y \quad \vee \quad (c \oplus d) \oplus x = \epsilon \oplus x$

26: $\neg\ c \oplus d = \epsilon$ by

23: $\neg\ \epsilon = d$

8: $\neg\ c \oplus d = \epsilon \quad \vee \quad \epsilon = d$

27: $c \oplus d = \epsilon$ by

25: $(c \oplus d) \oplus x = \epsilon \oplus x$

7: $\neg\ (c \oplus d) \oplus x = \epsilon \oplus x \quad \vee \quad c \oplus d = \epsilon$

28: QEA by

26: $\neg\ c \oplus d = \epsilon$

27: $c \oplus d = \epsilon$