Proof of Theorem 010

The theorem to be proved is

x begins with y & y begins with x $\rightarrow$ $x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x)$ begins with $(y)]$ & $[(y)$ begins with $(x)]$ & $[\neg\,(x) = (y)]]$

Special cases of the hypothesis and previous results:

0: x begins with y from H:x:y

1: y begins with x from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $\neg\, x$ begins with y $\vee$ $y \oplus c = x$ from $290^{\rightarrow}$;x;y:c

4: $\neg\, y$ begins with x $\vee$ $x \oplus d = y$ from $290^{\rightarrow}$;y;x:d

5: $x \oplus (d \oplus c) = (x \oplus d) \oplus c$ from 183;x;d;c

6: $x \oplus \epsilon = x$ from 196;x

7: $\neg\, x \oplus (d \oplus c) = x \oplus \epsilon$ $\vee$ $d \oplus c = \epsilon$ from 199;x;ϵ;$d \oplus c$

8: $\neg\, d \oplus c = \epsilon$ $\vee$ $\epsilon = d$ from 204;d;c

Equality substitutions:

9: $\neg\, y \oplus c = x$ $\vee$ $x \oplus \epsilon = y \oplus c$ $\vee$ $\neg\, x \oplus \epsilon = x$

10: $\neg\, x \oplus d = y$ $\vee$ $\neg\, x \oplus (d \oplus c) = (x \oplus d) \oplus c$ $\vee$ $x \oplus (d \oplus c) = (y) \oplus c$

11: $\neg\, x \oplus \epsilon = x$ $\vee$ $\neg\, x \oplus \epsilon = y$ $\vee$ $x = y$

12: $\neg\, \epsilon = d$ $\vee$ $x \oplus \epsilon = y$ $\vee$ $\neg\, x \oplus d = y$

13: $\neg\, x \oplus (d \oplus c) = y \oplus c$ $\vee$ $x \oplus (d \oplus c) = x \oplus \epsilon$ $\vee$ $\neg\, y \oplus c = x \oplus \epsilon$

Inferences:

14: $y \oplus c = x$ by

0: x begins with y

3: $\neg\, x$ begins with y $\vee$ $y \oplus c = x$

15: $x \oplus d = y$ by

1: y begins with x

4: $\neg$ y begins with x $\vee$ $x \oplus d = y$

16: $\neg\, x \oplus \epsilon = x \quad \vee \quad \neg\, x \oplus \epsilon = y$ by

2: $\neg\, y = x$

11: $\neg\, x \oplus \epsilon = x \quad \vee \quad \neg\, x \oplus \epsilon = y \quad \vee \quad y = x$

17: $\neg\, x \oplus d = y \quad \vee \quad x \oplus (d \oplus c) = y \oplus c$ by

5: $x \oplus (d \oplus c) = (x \oplus d) \oplus c$

10: $\neg\, x \oplus d = y \quad \vee \quad \neg\, x \oplus (d \oplus c) = (x \oplus d) \oplus c \quad \vee \quad x \oplus (d \oplus c) = y \oplus c$

18: $\neg\, y \oplus c = x \quad \vee \quad x \oplus \epsilon = y \oplus c$ by

6: $x \oplus \epsilon = x$

9: $\neg\, y \oplus c = x \quad \vee \quad x \oplus \epsilon = y \oplus c \quad \vee \quad \neg\, x \oplus \epsilon = x$

19: $\neg\, x \oplus \epsilon = y$ by

6: $x \oplus \epsilon = x$

16: $\neg\, x \oplus \epsilon = x \quad \vee \quad \neg\, x \oplus \epsilon = y$

20: $x \oplus \epsilon = y \oplus c$ by

14: $y \oplus c = x$

18: $\neg\, y \oplus c = x \quad \vee \quad x \oplus \epsilon = y \oplus c$

21: $\neg\, \epsilon = d \quad \vee \quad x \oplus \epsilon = y$ by

15: $x \oplus d = y$

12: $\neg\, \epsilon = d \quad \vee \quad x \oplus \epsilon = y \quad \vee \quad \neg\, x \oplus d = y$

22: $x \oplus (d \oplus c) = y \oplus c$ by

15: $x \oplus d = y$

17: $\neg\, x \oplus d = y \quad \vee \quad x \oplus (d \oplus c) = y \oplus c$

23: $\neg\, \epsilon = d$ by

19: $\neg\, x \oplus \epsilon = y$

21: $\neg\, \epsilon = d \quad \vee \quad x \oplus \epsilon = y$

24: $\neg\, x \oplus (d \oplus c) = y \oplus c \quad \vee \quad x \oplus (d \oplus c) = x \oplus \epsilon$ by

20: $x \oplus \epsilon = y \oplus c$

13: $\neg\, x \oplus (d \oplus c) = y \oplus c \quad \vee \quad x \oplus (d \oplus c) = x \oplus \epsilon \quad \vee \quad \neg\, x \oplus \epsilon = y \oplus c$

25: $x \oplus (d \oplus c) = x \oplus \epsilon$ by

22: $x \oplus (d \oplus c) = y \oplus c$

24: $\neg\, x \oplus (d \oplus c) = y \oplus c \quad \vee \quad x \oplus (d \oplus c) = x \oplus \epsilon$

26: $\neg\, d \oplus c = \epsilon$ by

23: $\neg\, \epsilon = d$

8: $\neg\, d \oplus c = \epsilon \quad \vee \quad \epsilon = d$

27: $d \oplus c = \epsilon$ by

25: $x \oplus (d \oplus c) = x \oplus \epsilon$

7: $\neg\, x \oplus (d \oplus c) = x \oplus \epsilon \quad \vee \quad d \oplus c = \epsilon$

28: QEA by

26: $\neg\, d \oplus c = \epsilon$

27: $d \oplus c = \epsilon$