# Proof of Theorem 008

The theorem to be proved is

$x \preceq \epsilon \quad \rightarrow \quad x = \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \preceq (\epsilon)] \quad \& \quad [\neg\, (x) = (\epsilon)]]$

## Special cases of the hypothesis and previous results:

0: $x \preceq \epsilon$ from H:$x$

1: $\neg\, \epsilon = x$ from H:$x$

2: $\neg\, x \preceq \epsilon \quad \vee \quad \text{Length}x \leq \text{Length}\epsilon$ from 264$^{\rightarrow}$;$x$;$\epsilon$

3: $\neg\, \text{Length}x \leq 0 \quad \vee \quad \text{Length}x = 0$ from 57;Length$x$

4: $\neg\, \text{Length}x = 0 \quad \vee \quad \epsilon = x$ from 001;$x$

5: $\text{Length}\epsilon = 0$ from 259

## Equality substitutions:

6: $\neg\, \text{Length}\epsilon = 0 \quad \vee \quad \neg\, \text{Length}x \leq \text{Length}\epsilon \quad \vee \quad \text{Length}x \leq 0$

## Inferences:

7: $\text{Length}x \leq \text{Length}\epsilon$ by
0: $x \preceq \epsilon$
2: $\neg\, x \preceq \epsilon \quad \vee \quad \text{Length}x \leq \text{Length}\epsilon$

8: $\neg\, \text{Length}x = 0$ by
1: $\neg\, \epsilon = x$
4: $\neg\, \text{Length}x = 0 \quad \vee \quad \epsilon = x$

9: $\neg\, \text{Length}x \leq \text{Length}\epsilon \quad \vee \quad \text{Length}x \leq 0$ by
5: $\text{Length}\epsilon = 0$
6: $\neg\, \text{Length}\epsilon = 0 \quad \vee \quad \neg\, \text{Length}x \leq \text{Length}\epsilon \quad \vee \quad \text{Length}x \leq 0$

10: $\text{Length}x \leq 0$ by
7: $\text{Length}x \leq \text{Length}\epsilon$
9: $\neg\, \text{Length}x \leq \text{Length}\epsilon \quad \vee \quad \text{Length}x \leq 0$

11: $\neg\ \mathrm{Length} x \leq 0$ by

8: $\neg\ \mathrm{Length} x = 0$

3: $\neg\ \mathrm{Length} x \leq 0 \quad \vee \quad \mathrm{Length} x = 0$

12: $QEA$ by

10: $\mathrm{Length} x \leq 0$

11: $\neg\ \mathrm{Length} x \leq 0$