Proof of Theorem 007

The theorem to be proved is

$a_1 \oplus c_1 = a_2 \oplus c_2 \quad \& \quad a_1 \preceq a_2 \quad \& \quad c_2 \neq \epsilon \quad \rightarrow \quad c_1 \neq \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(a_1 \oplus c_1) = (a_2 \oplus c_2)] \quad \& \quad [(a_1) \preceq (a_2)] \quad \& \quad [\neg\,(c_2) = (\epsilon)] \quad \& \quad [(c_1) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $a_2 \oplus c_2 = a_1 \oplus c_1$ from H:a_1:c_1:a_2:c_2

1: $a_1 \preceq a_2$ from H:a_1:c_1:a_2:c_2

2: $\neg\, \epsilon = c_2$ from H:a_1:c_1:a_2:c_2

3: $\epsilon = c_1$ from H:a_1:c_1:a_2:c_2

4: $\neg\, a_2 \oplus c_2 = a_1 \oplus c_1 \quad \vee \quad \neg\, a_1 \preceq a_2 \quad \vee \quad c_2 \preceq c_1$ from 006;a_1;c_1;a_2;c_2

5: $\neg\, c_2 \preceq \epsilon \quad \vee \quad \epsilon = c_2$ from 008;c_2

Equality substitutions:

6: $\neg\, \epsilon = c_1 \quad \vee \quad c_2 \preceq \epsilon \quad \vee \quad \neg\, c_2 \preceq c_1$

Inferences:

7: $\neg\, a_1 \preceq a_2 \quad \vee \quad c_2 \preceq c_1$ by
0: $a_2 \oplus c_2 = a_1 \oplus c_1$
4: $\neg\, a_2 \oplus c_2 = a_1 \oplus c_1 \quad \vee \quad \neg\, a_1 \preceq a_2 \quad \vee \quad c_2 \preceq c_1$

8: $c_2 \preceq c_1$ by
1: $a_1 \preceq a_2$
7: $\neg\, a_1 \preceq a_2 \quad \vee \quad c_2 \preceq c_1$

9: $\neg\, c_2 \preceq \epsilon$ by
2: $\neg\, \epsilon = c_2$
5: $\neg\, c_2 \preceq \epsilon \quad \vee \quad \epsilon = c_2$

10: $c_2 \preceq \epsilon \quad \vee \quad \neg\, c_2 \preceq c_1$ by
3: $\epsilon = c_1$
6: $\neg\, \epsilon = c_1 \quad \vee \quad c_2 \preceq \epsilon \quad \vee \quad \neg\, c_2 \preceq c_1$

11: $c_2 \preceq \epsilon$ by

8: $c_2 \preceq c_1$

10: $c_2 \preceq \epsilon \quad \vee \quad \neg\, c_2 \preceq c_1$

12: QEA by

9: $\neg\, c_2 \preceq \epsilon$

11: $c_2 \preceq \epsilon$