Proof of Theorem 006

The theorem to be proved is

$a_1 \oplus c_1 = a_2 \oplus c_2 \quad \& \quad a_1 \preceq a_2 \quad \rightarrow \quad c_2 \preceq c_1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(a_1 \oplus c_1) = (a_2 \oplus c_2)] \quad \& \quad [(a_1) \preceq (a_2)] \quad \& \quad [\neg (c_2) \preceq (c_1)]]$

Special cases of the hypothesis and previous results:

0: $a_2 \oplus c_2 = a_1 \oplus c_1$ from H:a_1:c_1:a_2:c_2

1: $a_1 \preceq a_2$ from H:a_1:c_1:a_2:c_2

2: $\neg\, c_2 \preceq c_1$ from H:a_1:c_1:a_2:c_2

3: $\neg\, a_1 \preceq a_2 \quad \vee \quad \text{Length}a_1 \leq \text{Length}a_2$ from $264^{\rightarrow}$;a_1;a_2

4: $c_2 \preceq c_1 \quad \vee \quad \neg\, \text{Length}c_2 \leq \text{Length}c_1$ from $264^{\leftarrow}$;c_2;c_1

5: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$ from 260;a_1;c_1

6: $(\text{Length}a_2) + (\text{Length}c_2) = \text{Length}(a_2 \oplus c_2)$ from 260;a_2;c_2

7: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, \text{Length}a_1 \leq \text{Length}a_2 \quad \vee \quad \text{Length}c_2 \leq \text{Length}c_1$ from 005;Lengtha_1;Lengthc_1;Lengtha_2;Lengthc_2

Equality substitutions:

8: $\neg\, a_2 \oplus c_2 = a_1 \oplus c_1 \quad \vee \quad (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$

9: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, \text{Length}(a_2 \oplus c_2) = (\text{Length}a_1) + (\text{Length}c_1)$

Inferences:

10: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$ by

0: $a_2 \oplus c_2 = a_1 \oplus c_1$

8: $\neg\, a_2 \oplus c_2 = a_1 \oplus c_1 \quad \vee \quad (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$

11: $\text{Length}a_1 \leq \text{Length}a_2$ by

1: $a_1 \preceq a_2$

3: $\neg\, a_1 \preceq a_2 \quad \vee \quad \text{Length}a_1 \leq \text{Length}a_2$

12: $\neg\, \text{Length}c_2 \leq \text{Length}c_1$ by

2: $\neg\, c_2 \preceq c_1$

4: $c_2 \preceq c_1 \quad \vee \quad \neg\, \text{Length}c_2 \leq \text{Length}c_1$

13: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2)$ by

5: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$

10: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_1 \oplus c_1)$

14: $(\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2)$ by

6: $(\text{Length}a_2) + (\text{Length}c_2) = \text{Length}(a_2 \oplus c_2)$

9: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = \text{Length}(a_2 \oplus c_2) \quad \vee \quad (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2)$

15: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \text{Length}c_2 \leq \text{Length}c_1$ by

11: $\text{Length}a_1 \leq \text{Length}a_2$

7: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, \text{Length}a_1 \leq \text{Length}a_2 \quad \vee \quad \text{Length}c_2 \leq \text{Length}c_1$

16: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1)$ by

12: $\neg\, \text{Length}c_2 \leq \text{Length}c_1$

15: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \text{Length}c_2 \leq \text{Length}c_1$

17: $(\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1)$ by

13: $(\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2)$

14: $(\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1) \quad \vee \quad \neg\, (\text{Length}a_1) + (\text{Length}c_1) = \text{Length}(a_2 \oplus c_2)$

18: QEA by

16: $\neg\, (\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1)$

17: $(\text{Length}a_2) + (\text{Length}c_2) = (\text{Length}a_1) + (\text{Length}c_1)$