Proof of Theorem 005

The theorem to be proved is

$x_1 + y_1 = x_2 + y_2 \quad \& \quad x_1 \le x_2 \quad \to \quad y_2 \le y_1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x_1 + y_1) = (x_2 + y_2)] \quad \& \quad [(x_1) \le (x_2)] \quad \& \quad [\neg\, (y_2) \le (y_1)]]$

Special cases of the hypothesis and previous results:

0: $x_2 + y_2 = x_1 + y_1$ from H:x_1:y_1:x_2:y_2

1: $x_1 \le x_2$ from H:x_1:y_1:x_2:y_2

2: $\neg\, y_2 \le y_1$ from H:x_1:y_1:x_2:y_2

3: $\neg\, x_1 \le x_2 \quad \vee \quad x_1 + (x_2 - x_1) = x_2$ from 68;x_1;x_2

4: $x_1 + ((x_2 - x_1) + y_2) = (x_1 + (x_2 - x_1)) + y_2$ from 72;x_1;$x_2 - x_1$;y_2

5: $\neg\, x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad (x_2 - x_1) + y_2 = y_1$ from 120;x_1;y_1;$(x_2 - x_1) + y_2$

6: $y_2 + (x_2 - x_1) = (x_2 - x_1) + y_2$ from 98;$x_2 - x_1$;y_2

7: $y_2 \le y_2 + (x_2 - x_1)$ from 71;y_2;$x_2 - x_1$

Equality substitutions:

8: $\neg\, x_1 + (x_2 - x_1) = x_2 \quad \vee \quad (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1 \quad \vee \quad \neg\, (x_2) + y_2 = x_1 + y_1$

9: $\neg\, x_1 + ((x_2 - x_1) + y_2) = (x_1 + (x_2 - x_1)) + y_2 \quad \vee \quad x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad \neg\, (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$

10: $\neg\, (x_2 - x_1) + y_2 = y_1 \quad \vee \quad \neg\, y_2 + (x_2 - x_1) = (x_2 - x_1) + y_2 \quad \vee \quad y_2 + (x_2 - x_1) = y_1$

11: $\neg\, y_2 + (x_2 - x_1) = y_1 \quad \vee \quad \neg\, y_2 \le y_2 + (x_2 - x_1) \quad \vee \quad y_2 \le y_1$

Inferences:

12: $\neg\, x_1 + (x_2 - x_1) = x_2 \quad \vee \quad (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$ by

0: $x_2 + y_2 = x_1 + y_1$

8: $\neg\, x_1 + (x_2 - x_1) = x_2 \quad \vee \quad (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1 \quad \vee \quad \neg\, x_2 + y_2 = x_1 + y_1$

13: $x_1 + (x_2 - x_1) = x_2$ by

1: $x_1 \leq x_2$

3: $\neg\, x_1 \leq x_2 \quad \vee \quad x_1 + (x_2 - x_1) = x_2$

14: $\neg\, y_2 + (x_2 - x_1) = y_1 \quad \vee \quad \neg\, y_2 \leq y_2 + (x_2 - x_1)$ by

2: $\neg\, y_2 \leq y_1$

11: $\neg\, y_2 + (x_2 - x_1) = y_1 \quad \vee \quad \neg\, y_2 \leq y_2 + (x_2 - x_1) \quad \vee \quad y_2 \leq y_1$

15: $x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad \neg\, (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$ by

4: $x_1 + ((x_2 - x_1) + y_2) = (x_1 + (x_2 - x_1)) + y_2$

9: $\neg\, x_1 + ((x_2 - x_1) + y_2) = (x_1 + (x_2 - x_1)) + y_2 \quad \vee \quad x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad \neg\, (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$

16: $\neg\, (x_2 - x_1) + y_2 = y_1 \quad \vee \quad y_2 + (x_2 - x_1) = y_1$ by

6: $y_2 + (x_2 - x_1) = (x_2 - x_1) + y_2$

10: $\neg\, (x_2 - x_1) + y_2 = y_1 \quad \vee \quad \neg\, y_2 + (x_2 - x_1) = (x_2 - x_1) + y_2 \quad \vee \quad y_2 + (x_2 - x_1) = y_1$

17: $\neg\, y_2 + (x_2 - x_1) = y_1$ by

7: $y_2 \leq y_2 + (x_2 - x_1)$

14: $\neg\, y_2 + (x_2 - x_1) = y_1 \quad \vee \quad \neg\, y_2 \leq y_2 + (x_2 - x_1)$

18: $(x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$ by

13: $x_1 + (x_2 - x_1) = x_2$

12: $\neg\, x_1 + (x_2 - x_1) = x_2 \quad \vee \quad (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$

19: $\neg\, (x_2 - x_1) + y_2 = y_1$ by

17: $\neg\, y_2 + (x_2 - x_1) = y_1$

16: $\neg\, (x_2 - x_1) + y_2 = y_1 \quad \vee \quad y_2 + (x_2 - x_1) = y_1$

20: $x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1$ by

18: $(x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$

15: $x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad \neg\, (x_1 + (x_2 - x_1)) + y_2 = x_1 + y_1$

21: $\neg\, x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1$ by

19: $\neg\, (x_2 - x_1) + y_2 = y_1$

5: $\neg\, x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1 \quad \vee \quad (x_2 - x_1) + y_2 = y_1$

22: QEA by

20: $x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1$

21: $\neg\, x_1 + ((x_2 - x_1) + y_2) = x_1 + y_1$