Proof of Theorem 003b

The theorem to be proved is

ϵ begins with a_1 & ϵ begins with a_2 & $a_1 \preceq a_2$ $\rightarrow$ a_2 begins with a_1

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[(ϵ) begins with (a_1)] & [(ϵ) begins with (a_2)] & [(a_1) $\preceq$ (a_2)] & [$\neg$ (a_2) begins with (a_1)]]

Special cases of the hypothesis and previous results:

0: ϵ begins with a_1 from H:a_1:a_2

1: ϵ begins with a_2 from H:a_1:a_2

2: $\neg$ a_2 begins with a_1 from H:a_1:a_2

3: $\neg$ ϵ begins with a_2 $\vee$ $\epsilon = a_2$ from 002;a_2

Equality substitutions:

4: $\neg$ $\epsilon = a_2$ $\vee$ $\neg$ ϵ begins with a_1 $\vee$ a_2 begins with a_1

Inferences:

5: $\neg$ $\epsilon = a_2$ $\vee$ a_2 begins with a_1 by
0: ϵ begins with a_1
4: $\neg$ $\epsilon = a_2$ $\vee$ $\neg$ ϵ begins with a_1 $\vee$ a_2 begins with a_1

6: $\epsilon = a_2$ by
1: ϵ begins with a_2
3: $\neg$ ϵ begins with a_2 $\vee$ $\epsilon = a_2$

7: $\neg$ $\epsilon = a_2$ by
2: $\neg$ a_2 begins with a_1
5: $\neg$ $\epsilon = a_2$ $\vee$ a_2 begins with a_1

8: QEA by
6: $\epsilon = a_2$
7: $\neg$ $\epsilon = a_2$