Proof of Theorem 002

The theorem to be proved is

ϵ begins with x $\vee$ ϵ ends with x $\rightarrow$ $x = \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\epsilon)$ begins with (x) $\vee$ (ϵ) ends with $(x)]$ & $[\neg\, (x) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: ϵ begins with x $\vee$ ϵ ends with x from H:x

1: $\neg\, \epsilon = x$ from H:x

2: $\neg\, \epsilon$ begins with x $\vee$ $x \oplus c = \epsilon$ from 290$^{\rightarrow}$;ϵ;x:c

3: $\neg\, \epsilon$ ends with x $\vee$ $a \oplus x = \epsilon$ from 288$^{\rightarrow}$;ϵ;x:a

4: $\neg\, x \oplus c = \epsilon$ $\vee$ $\epsilon = x$ from 204;x;c

5: $\neg\, a \oplus x = \epsilon$ $\vee$ $\epsilon = x$ from 204;a;x

Inferences:

6: $\neg\, x \oplus c = \epsilon$ by

1: $\neg\, \epsilon = x$

4: $\neg\, x \oplus c = \epsilon$ $\vee$ $\epsilon = x$

7: $\neg\, a \oplus x = \epsilon$ by

1: $\neg\, \epsilon = x$

5: $\neg\, a \oplus x = \epsilon$ $\vee$ $\epsilon = x$

8: $\neg\, \epsilon$ begins with x by

6: $\neg\, x \oplus c = \epsilon$

2: $\neg\, \epsilon$ begins with x $\vee$ $x \oplus c = \epsilon$

9: $\neg\, \epsilon$ ends with x by

7: $\neg\, a \oplus x = \epsilon$

3: $\neg\, \epsilon$ ends with x $\vee$ $a \oplus x = \epsilon$

10: ϵ ends with x by

8: $\neg\, \epsilon$ begins with x

0: ϵ begins with x $\vee$ ϵ ends with x

11: QEA by

9: $\neg\, \epsilon$ ends with x

10: ϵ ends with x