

Proof of Theorem 001

The theorem to be proved is

$$\text{Length } x = 0 \quad \rightarrow \quad x = \epsilon$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[(\text{Length}x) = (0)] \quad \& \quad [\neg (x) = (\epsilon)]]$$

Special cases of the hypothesis and previous results:

- 0: $\text{Length}x = 0$ from $H:x$
- 1: $\neg \epsilon = x$ from $H:x$
- 2: $\epsilon = x \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$ from [253](#);x
- 3: $(\text{Length}(\text{Chop}x)) + (\text{Length}\underline{0}) = \text{Length}((\text{Chop}x) \oplus \underline{0})$ from [260](#);Chopx;0
- 4: $(\text{Length}(\text{Chop}x)) + (\text{Length}\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{1})$ from [260](#);Chopx;1
- 5: $\text{Length}\underline{0} = 1$ from [278](#)
- 6: $\text{Length}\underline{1} = 1$ from [278](#)
- 7: $\neg (\text{Length}(\text{Chop}x)) + 1 = 0 \quad \vee \quad 1 = 0$ from [15](#);Length(Chopx);1
- 8: $S0 = 1$ from [115](#)
- 9: $\neg S0 = 0$ from [3](#);0

Equality substitutions:

- 10: $\neg \text{Length}x = 0 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = 0$
- 11: $\neg (\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad \neg \text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1$
 $\vee \quad \text{Length}(x) = (\text{Length}(\text{Chop}x)) + 1$
- 12: $\neg (\text{Chop}x) \oplus \underline{1} = x \quad \vee \quad \neg \text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1$
 $\vee \quad \text{Length}(x) = (\text{Length}(\text{Chop}x)) + 1$
- 13: $\neg \text{Length}\underline{0} = 1 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + (\text{Length}\underline{0}) = \text{Length}((\text{Chop}x) \oplus \underline{0})$
 $\vee \quad (\text{Length}(\text{Chop}x)) + (\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{0})$
- 14: $\neg \text{Length}\underline{1} = 1 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + (\text{Length}\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{1})$
 $\vee \quad (\text{Length}(\text{Chop}x)) + (\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{1})$
- 15: $\neg 1 = 0 \quad \vee \quad \neg S0 = \underline{1} \quad \vee \quad S0 = 0$

Inferences:

- 16: $\neg (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = 0 \quad \text{by}$
0: $\text{Length}x = 0$
10: $\neg \text{Length}x = 0 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = 0$
- 17: $(\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x \quad \text{by}$
1: $\neg \epsilon = x$
2: $\epsilon = x \quad \vee \quad (\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$
- 18: $\neg \text{Length}\underline{0} = 1 \quad \vee \quad \text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1 \quad \text{by}$
3: $(\text{Length}(\text{Chop}x)) + (\text{Length}\underline{0}) = \text{Length}((\text{Chop}x) \oplus \underline{0})$
13: $\neg \text{Length}\underline{0} = 1 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + (\text{Length}\underline{0}) = \text{Length}((\text{Chop}x) \oplus \underline{0})$
 $\vee \quad \text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1$
- 19: $\neg \text{Length}\underline{1} = 1 \quad \vee \quad \text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1 \quad \text{by}$
4: $(\text{Length}(\text{Chop}x)) + (\text{Length}\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{1})$
14: $\neg \text{Length}\underline{1} = 1 \quad \vee \quad \neg (\text{Length}(\text{Chop}x)) + (\text{Length}\underline{1}) = \text{Length}((\text{Chop}x) \oplus \underline{1})$
 $\vee \quad \text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1$
- 20: $\text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1 \quad \text{by}$
5: $\text{Length}\underline{0} = 1$
18: $\neg \text{Length}\underline{0} = 1 \quad \vee \quad \text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1$
- 21: $\text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1 \quad \text{by}$
6: $\text{Length}\underline{1} = 1$
19: $\neg \text{Length}\underline{1} = 1 \quad \vee \quad \text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1$
- 22: $\neg 1 = 0 \quad \vee \quad S0 = 0 \quad \text{by}$
8: $S0 = 1$
15: $\neg 1 = 0 \quad \vee \quad \neg S0 = 1 \quad \vee \quad S0 = 0$
- 23: $\neg 1 = 0 \quad \text{by}$
9: $\neg S0 = 0$
22: $\neg 1 = 0 \quad \vee \quad S0 = 0$
- 24: $\neg (\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x \quad \text{by}$
20: $\text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1$
11: $\neg (\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad \neg \text{Length}((\text{Chop}x) \oplus \underline{0}) = (\text{Length}(\text{Chop}x)) + 1$
 $\vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$

- 25: $\neg(\text{Chop}x) \oplus \underline{1} = x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$ by
 21: $\text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1$
 12: $\neg(\text{Chop}x) \oplus \underline{1} = x \quad \vee \quad \neg \text{Length}((\text{Chop}x) \oplus \underline{1}) = (\text{Length}(\text{Chop}x)) + 1$
 $\vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$
- 26: $\neg(\text{Length}(\text{Chop}x)) + 1 = 0$ by
 23: $\neg 1 = 0$
 7: $\neg(\text{Length}(\text{Chop}x)) + 1 = 0 \quad \vee \quad 1 = 0$
- 27: $\neg(\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$ by
 26: $\neg(\text{Length}(\text{Chop}x)) + 1 = 0$
 16: $\neg(\text{Length}(\text{Chop}x)) + 1 = \text{Length}x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = 0$
- 28: $\neg(\text{Chop}x) \oplus \underline{0} = x$ by
 27: $\neg(\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$
 24: $\neg(\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$
- 29: $\neg(\text{Chop}x) \oplus \underline{1} = x$ by
 27: $\neg(\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$
 25: $\neg(\text{Chop}x) \oplus \underline{1} = x \quad \vee \quad (\text{Length}(\text{Chop}x)) + 1 = \text{Length}x$
- 30: $(\text{Chop}x) \oplus \underline{1} = x$ by
 28: $\neg(\text{Chop}x) \oplus \underline{0} = x$
 17: $(\text{Chop}x) \oplus \underline{0} = x \quad \vee \quad (\text{Chop}x) \oplus \underline{1} = x$
- 31: *QEA* by
 29: $\neg(\text{Chop}x) \oplus \underline{1} = x$
 30: $(\text{Chop}x) \oplus \underline{1} = x$