Proof of Theorem 000b

The theorem to be proved is

$x_1 \oplus \epsilon = x_2 \oplus \epsilon \quad \rightarrow \quad x_1 = x_2$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x_1 \oplus \epsilon) = (x_2 \oplus \epsilon)] \quad \& \quad [\neg\,(x_1) = (x_2)]]$

Special cases of the hypothesis and previous results:

0: $x_2 \oplus \epsilon = x_1 \oplus \epsilon$ from H:x_1:x_2

1: $\neg\, x_2 = x_1$ from H:x_1:x_2

2: $x_1 \oplus \epsilon = x_1$ from 196;x_1

3: $x_2 \oplus \epsilon = x_2$ from 196;x_2

Equality substitutions:

4: $\neg\, x_2 \oplus \epsilon = x_1 \oplus \epsilon \quad \vee \quad x_2 \oplus \epsilon = x_1 \quad \vee \quad \neg\, x_1 \oplus \epsilon = x_1$

5: $\neg\, x_2 \oplus \epsilon = x_2 \quad \vee \quad \neg\, x_2 \oplus \epsilon = x_1 \quad \vee \quad x_2 = x_1$

Inferences:

6: $x_2 \oplus \epsilon = x_1 \quad \vee \quad \neg\, x_1 \oplus \epsilon = x_1$ by
0: $x_2 \oplus \epsilon = x_1 \oplus \epsilon$
4: $\neg\, x_2 \oplus \epsilon = x_1 \oplus \epsilon \quad \vee \quad x_2 \oplus \epsilon = x_1 \quad \vee \quad \neg\, x_1 \oplus \epsilon = x_1$

7: $\neg\, x_2 \oplus \epsilon = x_2 \quad \vee \quad \neg\, x_2 \oplus \epsilon = x_1$ by
1: $\neg\, x_2 = x_1$
5: $\neg\, x_2 \oplus \epsilon = x_2 \quad \vee \quad \neg\, x_2 \oplus \epsilon = x_1 \quad \vee \quad x_2 = x_1$

8: $x_2 \oplus \epsilon = x_1$ by
2: $x_1 \oplus \epsilon = x_1$
6: $x_2 \oplus \epsilon = x_1 \quad \vee \quad \neg\, x_1 \oplus \epsilon = x_1$

9: $\neg\, x_2 \oplus \epsilon = x_1$ by
3: $x_2 \oplus \epsilon = x_2$
7: $\neg\, x_2 \oplus \epsilon = x_2 \quad \vee \quad \neg\, x_2 \oplus \epsilon = x_1$

10: QEA by
8: $x_2 \oplus \epsilon = x_1$
9: $\neg\, x_2 \oplus \epsilon = x_1$