Mat104 Geometric Series, Sequences and L'Hôpital's Rule from Old Exams

- (1) For which values of x does $\sum_{n=0}^{\infty} e^{nx}$ converge? What is the value when it converges?
- (2) Evaluate $\sum_{n=4}^{\infty} \left(-\frac{2}{3}\right)^n$. (3) Evaluate $\sum_{n=0}^{\infty} \frac{2^n + 3^{n+1} + 4^{n+2}}{5^n}$. (4) Find $2 + \sum_{n=1}^{\infty} \frac{2 + (-3)^n}{5^n}$. (5) Evaluate $\sum_{n=0}^{\infty} \frac{2 + 2^n}{4^n}$. (6) Find $\lim_{n \to \infty} \frac{\ln(n^2 + n)}{\ln(n^2 - n)}$ or show that it does not exist. (7) Evaluate or show that $\lim_{n \to \infty} \frac{3n^3 + 1}{n^4 + n^2 + n + 8}$ does not exist: (8) Find each $\lim_{n \to \infty} \frac{n + 17 \arctan(n) + 2}{1 - n}$ or show that it does not exist. (9) Find $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{2^n}$ or show that it does not exist. (10) Find the following limits:

(a)
$$\lim_{n \to \infty} \frac{\left(1 + \frac{1}{n}\right)^n}{3\sqrt{n}}$$
 (b) $\lim_{n \to \infty} \frac{\ln(n+1000)}{\ln(n^2)}$