MATH 104 - FINAL EXAM
Friday, January 17, 2003, 8:30AM-11:30AM
McCosh 50

Note: Average was approximately 60 percent. Considered hard but fair.

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1. *(8 points)* Compute the following integrals:

   (a) $\int \frac{x}{(1-x^2)^{3/2}}\,dx$

   (b) $\int x\ln(x+1)\,dx$

2. *(12 points)*

   (a) Let R be the region bounded by the x-axis and the graph of $y = 1/(x^4+1)$ as x runs from 0 to ∞. Find the volume of the solid of revolution obtained by revolving R about the y-axis.

   (b) Calculate the area of the surface obtained by revolving the graph of $y = e^x$ between the points $(0,1)$ and $(1,e)$ around the x-axis.

3. *(16 points)* Determine whether the following integrals converge or diverge. Give your reasons.

   (a) $\int_0^\infty \frac{dx}{\sqrt{x}+x^3}$

   (b) $\int_0^1 \frac{\tan\sqrt{x}}{x+x^2}\,dx$

   (c) $\int_0^1 \frac{\ln(1+x)}{x^3}\,dx$

   (d) $\int_1^\infty \frac{dx}{x\ln x}$

4. *(16 points)* Determine whether the following series converge or diverge. Give your reasons.

   (a) $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5+1}}$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^3+1}$

   (c) $\sum_{n=0}^{\infty} \frac{n^2\cdot 3^n}{n!}$

   (d) $\sum_{n=0}^{\infty} \left(\frac{n+1}{n+3}\right)^{n^2}$

5. *(12 points)* Let $f(x) = \sum_{n=0}^{\infty} \frac{1}{n+2}\left(\frac{x-2}{3}\right)^n$.

   (a) For what values of x does the series converge? Give your reasons.

   (b) Find $f^{(50)}(2)$.

6. *(12 points)*

   (a) Use Taylor series to compute $\lim_{x\to 0} \frac{(e^x - 1 - x)^2 \cos x}{x(\sin x - x)}$.

   (b) Find the Taylor series of $F(x) = \int_0^x \frac{dt}{1+t^4}$ centered at $x = 0$. For what values of x does it converge?

7. *(12 points)* For the questions below express your answers in the form $a+ib$ where a and b are real numbers. Simplify your expressions for a and b.

   (a) Simplify $\left(\frac{7+i}{3+4i}\right)^{43}$.

   (b) Solve $z^4 = -8iz$.

8. *(12 points)* Find all real solutions to the following differential equations.

   (a) $y'' + 2y' + 10y = 0$

   (b) $2y'' + y' - 3y = 0$