IS THE MULTIDIMENSIONAL ODD SHIFT SOFIC?

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Interpret a configuration $x \in \{0,1\}^{\mathbb{Z}^2}$ as a percolation, i.e. as defining a subset of $\mathbb{Z}^d$ (the 1's) which in turn induces a subgraph of $\mathbb{Z}^d$, where as edges we take points whose ℓ^1-distance is 1 (this is the same as taking the Cayley graph of $\mathbb{Z}^d$ with the standard generating set). We can now talk about the connected components of x.

Let $X \subseteq \{0,1\}^{\mathbb{Z}^2}$ be the set of configurations in which every finite component has even size, and Y the set of configurations of odd size. It is an exercise to show that X is sofic.

Problem. Is Y sofic?

I believe that it is not, but I have no idea how to prove it.