A RATIO ERGODIC THEOREM FOR GROUPS

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There is a classical ergodic theorem, due to Hopf, for a transformation T acting ergodically on a σ-finite measure space: for $f, g \in L^1$,

$$\frac{\sum_{i=-n}^{n} T^i f}{\sum_{i=-n}^{n} T^i g} \to \frac{....f}{...g}$$

almost everywhere.

Recently I proved this for $\mathbb{Z}^d$ actions, where the sum is taken over balls in an arbitrary norm (for "one-sided" averages in which the sum is over $[0, n]^d$ the theorem is false; this was shown by Krengel and Brunel. The $\mathbb{Z}^d$ case was proved previously by Jack Feldman under the hypothesis that the generators of the action are conservative).

Problem. Is there any similar result for actions of non-abelian groups (e.g. groups with polynomial growth?).

In [??] I showed that in many non-abelian settings (e.g. the discrete Heisenberg group) the maximal inequality is false for these ratios, at lease when the sum is over some natural families of sets. So this may be a hard questions, in that the standard method – proving convergence for a dense family of functions and extending to all of L^1 using the maximal inequality – cannot work.