GENERICITY AND APPROXIMATION FOR CONNECTED SYSTEMS

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In my paper *Genericity in topological dynamics* (Ergodic Theory Dynamical Systems, 2008, Vol. 28 pp. 125-165) I studied the space $\mathcal{S}$ of all topological dynamical systems (on compact metric spaces) by considering the subshifts of the full shift over the Hilbert cube, and putting the Hausdorff metric on this space. This study was motivated by classical questions about measure preserving systems, which can be parameterize either by fixing the measure space and putting a topology on the group of measure-preserving self-maps of the space, or by fixing the space to be the full shift and considering the space of invariant measures. As it turns out, the topological model above is similarly equivalent to the space of homeomorphisms of the Cantor set (this is a dynamical version of the classical fact that in any perfect compact metric space X, the Cantor sets are dense in the space of closed subsets of X).

In particular, generically a topological dynamical system is totally disconnected, and most of the results about $\mathcal{S}$ are for that case. Little is known about the space of connected systems. Here is a basic question:

Problem. Consider the space of topologically weak mixing systems with *connected* phase spaces.

(1) Are the uniformly rigid systems generic?
(2) Does every minimal system have dense isomorphism class?

Both weak mixing, connectivity and rigidity are G_δ-conditions so the spaces in (1) are Polish. If we drop the connectedness condition, then totally disconnected systems are generic among the weak mixing systems, and since the only rigid systems of this sort are equicontinuous they are not weakly mixing; so the answer to (1) is negative. On the other hand, (2) is then true. Neither of the arguments works in the connected case.